

Filorames amb GGB

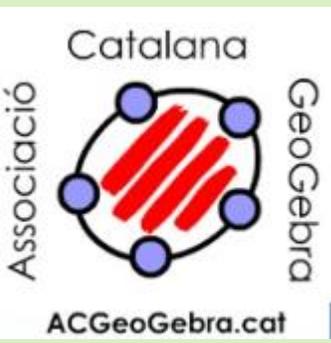
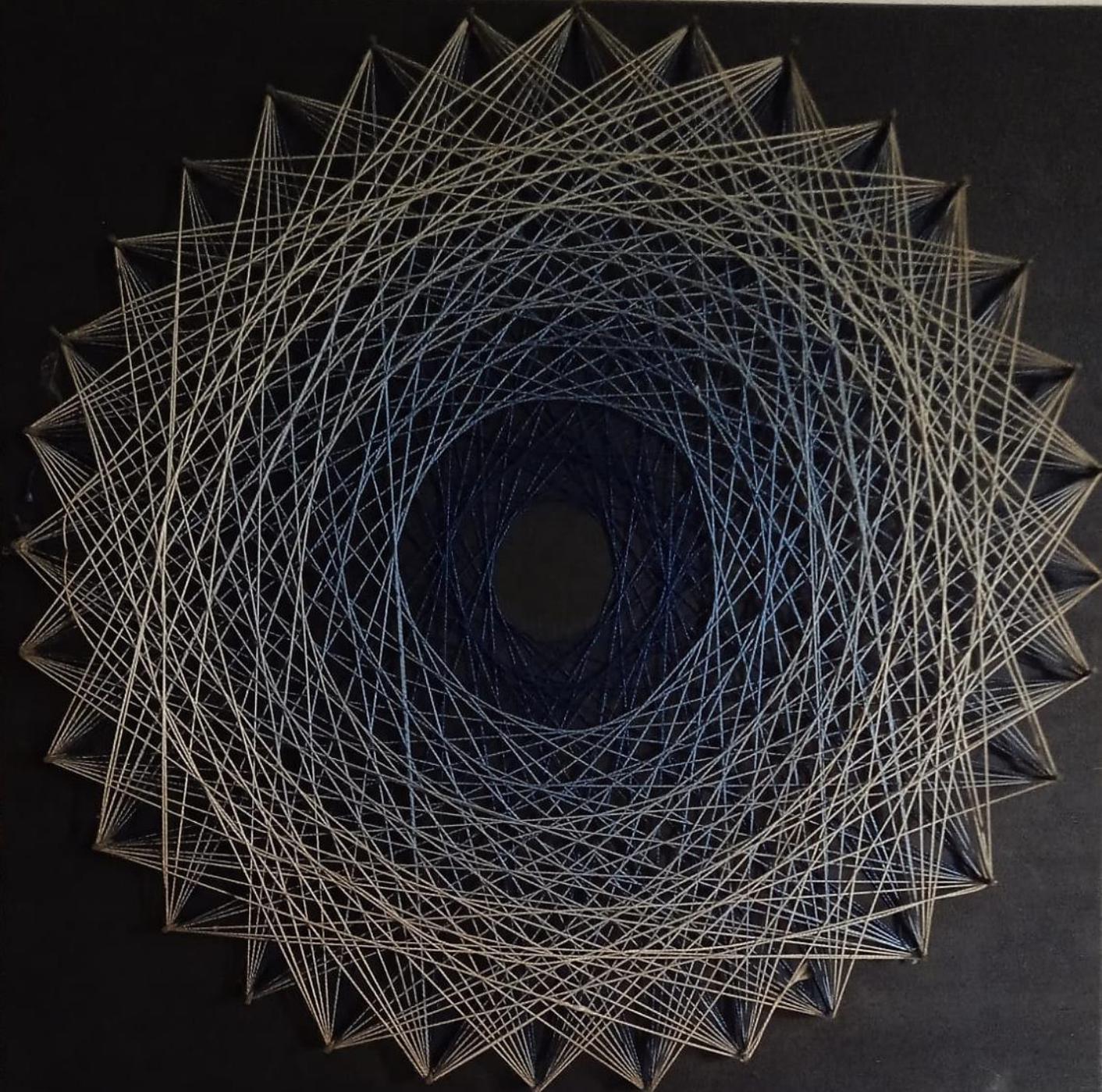
Una experiència per
treballar el sentit
algebraic a 2nESO

Barcelona, 22.02.2025

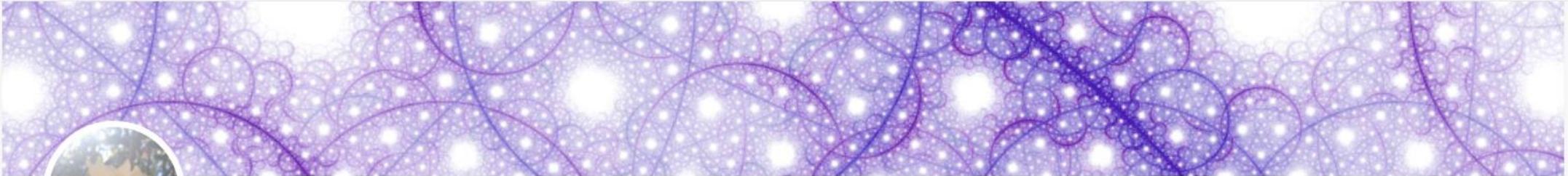
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INEE

UCLM



**Federación
Española de
Sociedades de
Profesores de
Matemáticas**



Elena Gajate Paniagua



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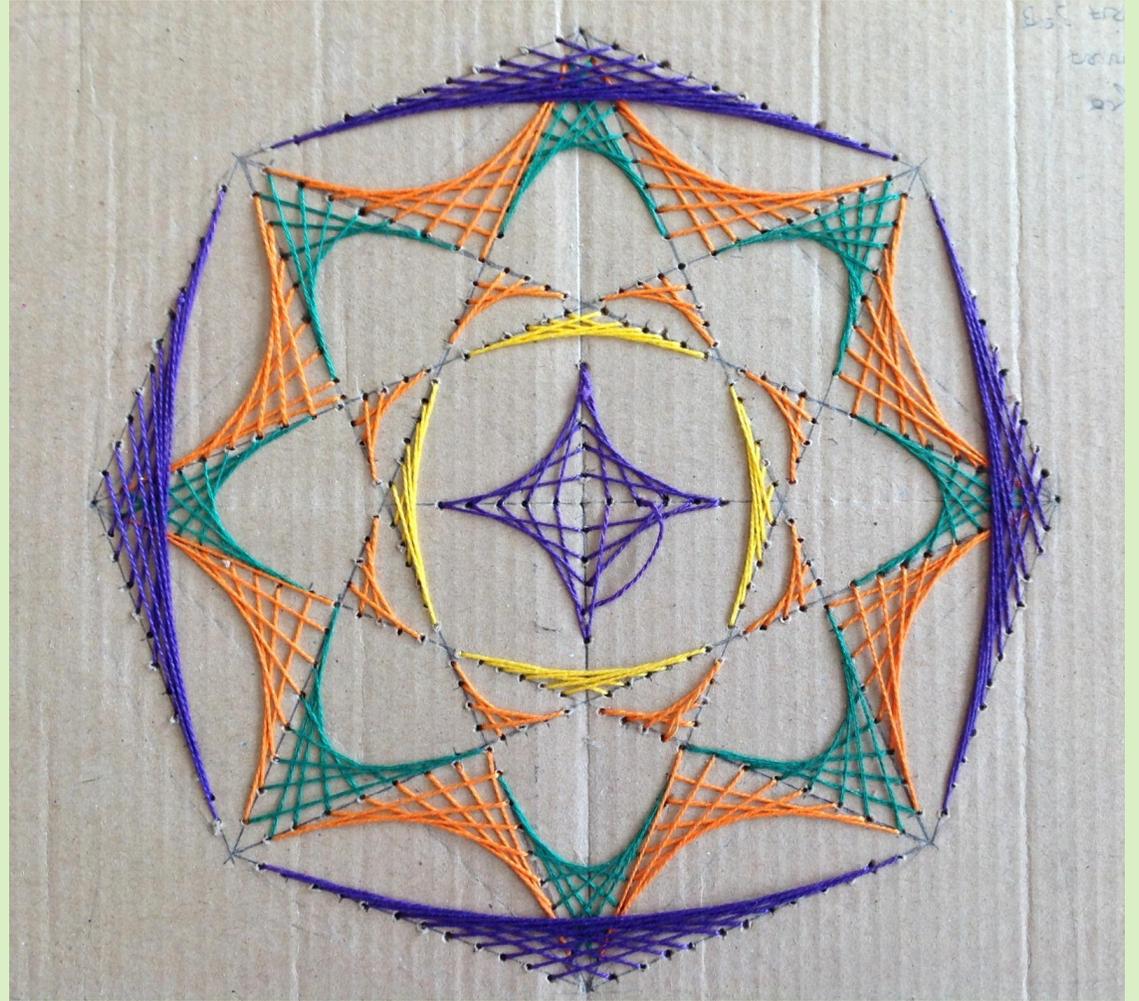
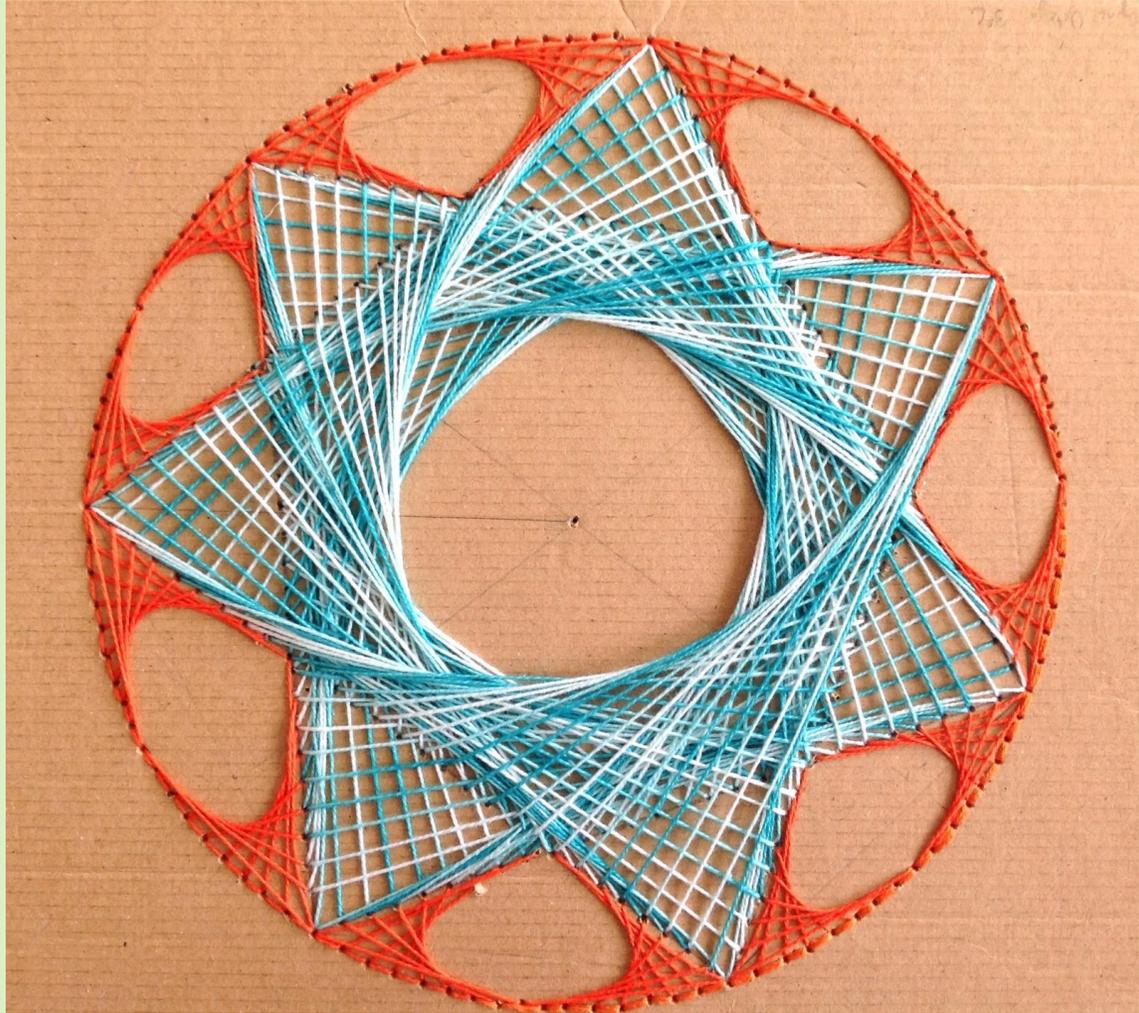
- RECURSOS**
- FAVORITOS
- PERSONAS

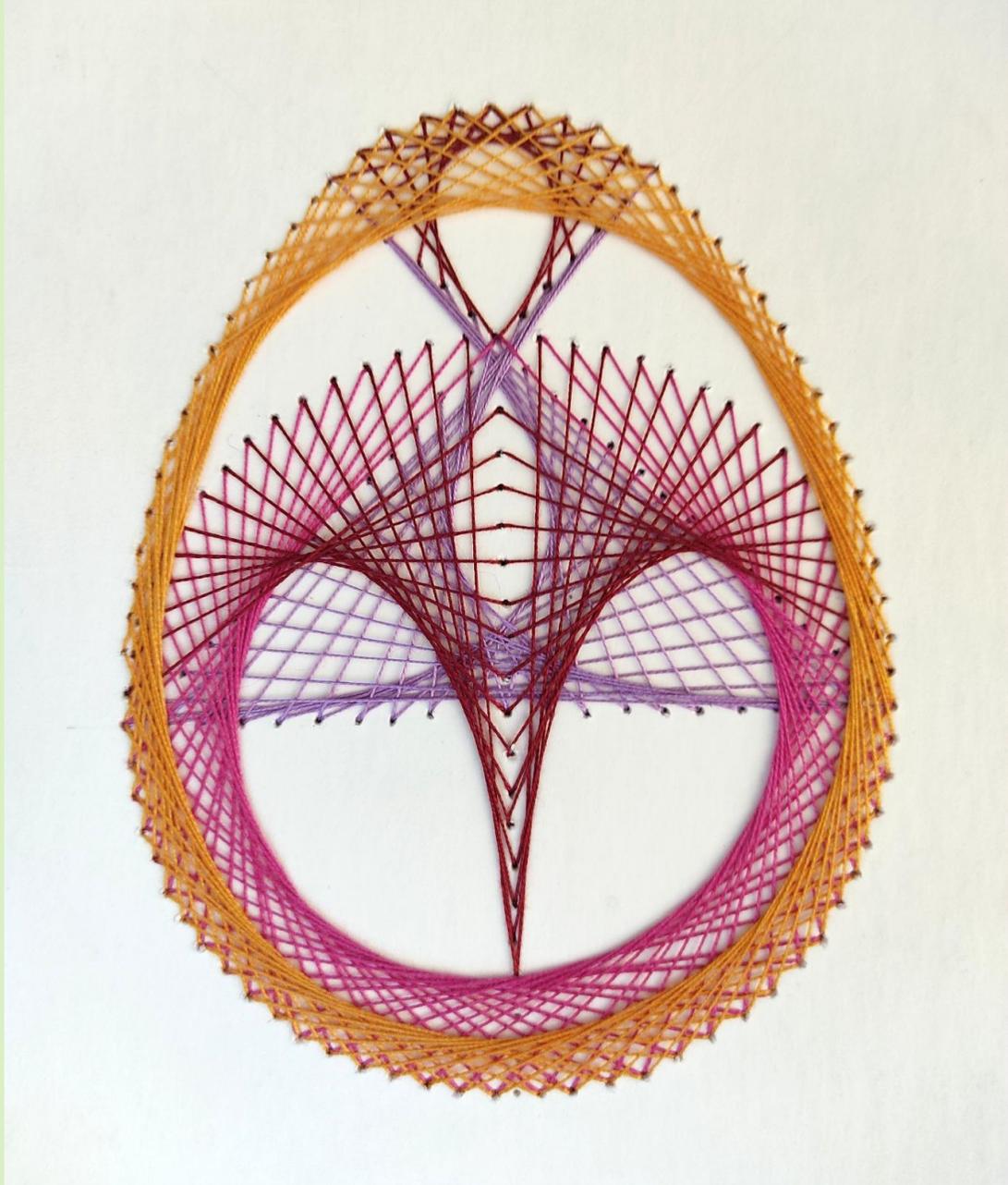
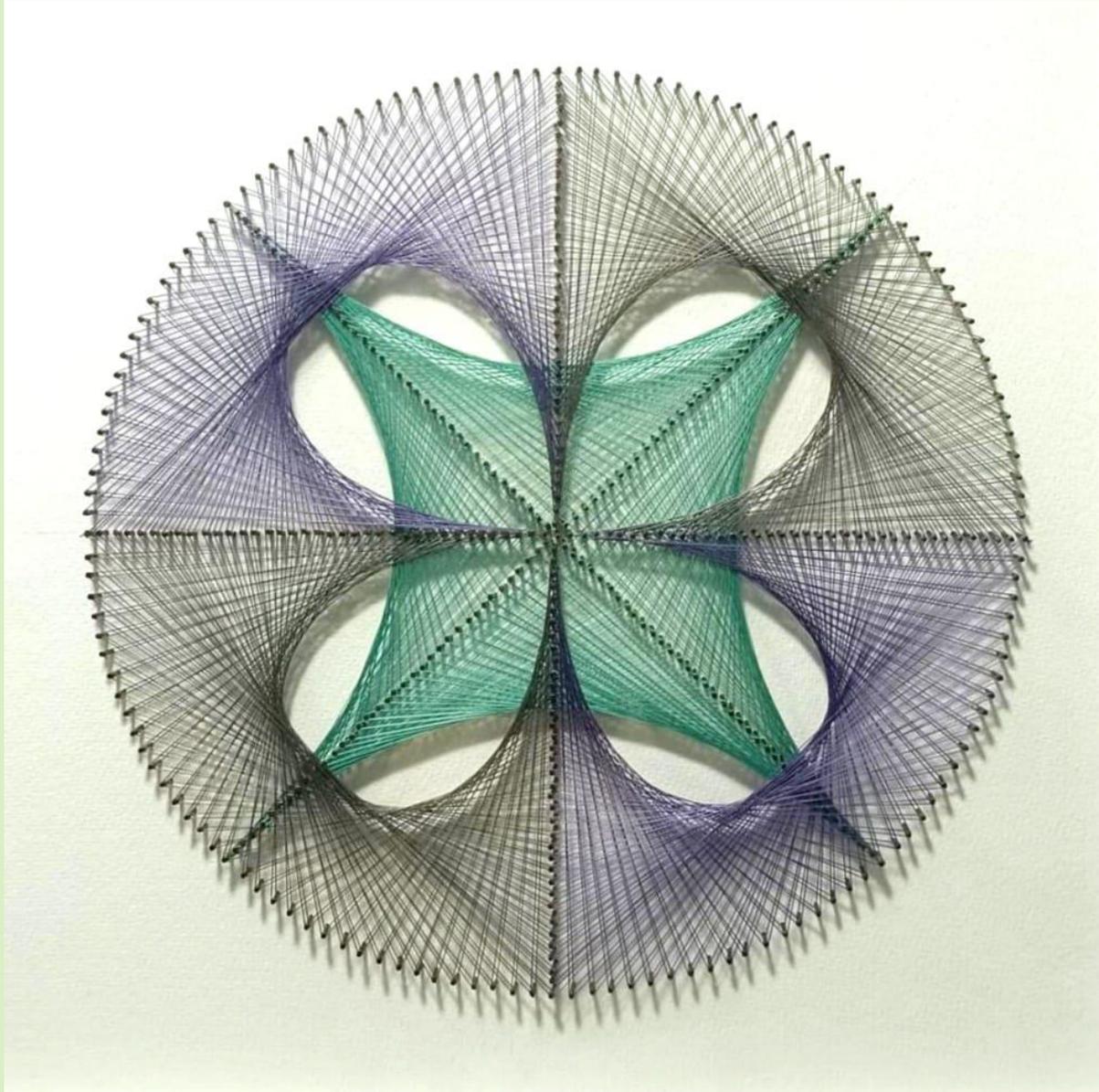
+ CREAR

Carpetas

Última modificación ▼ Recurso de cualquier tipo ▼

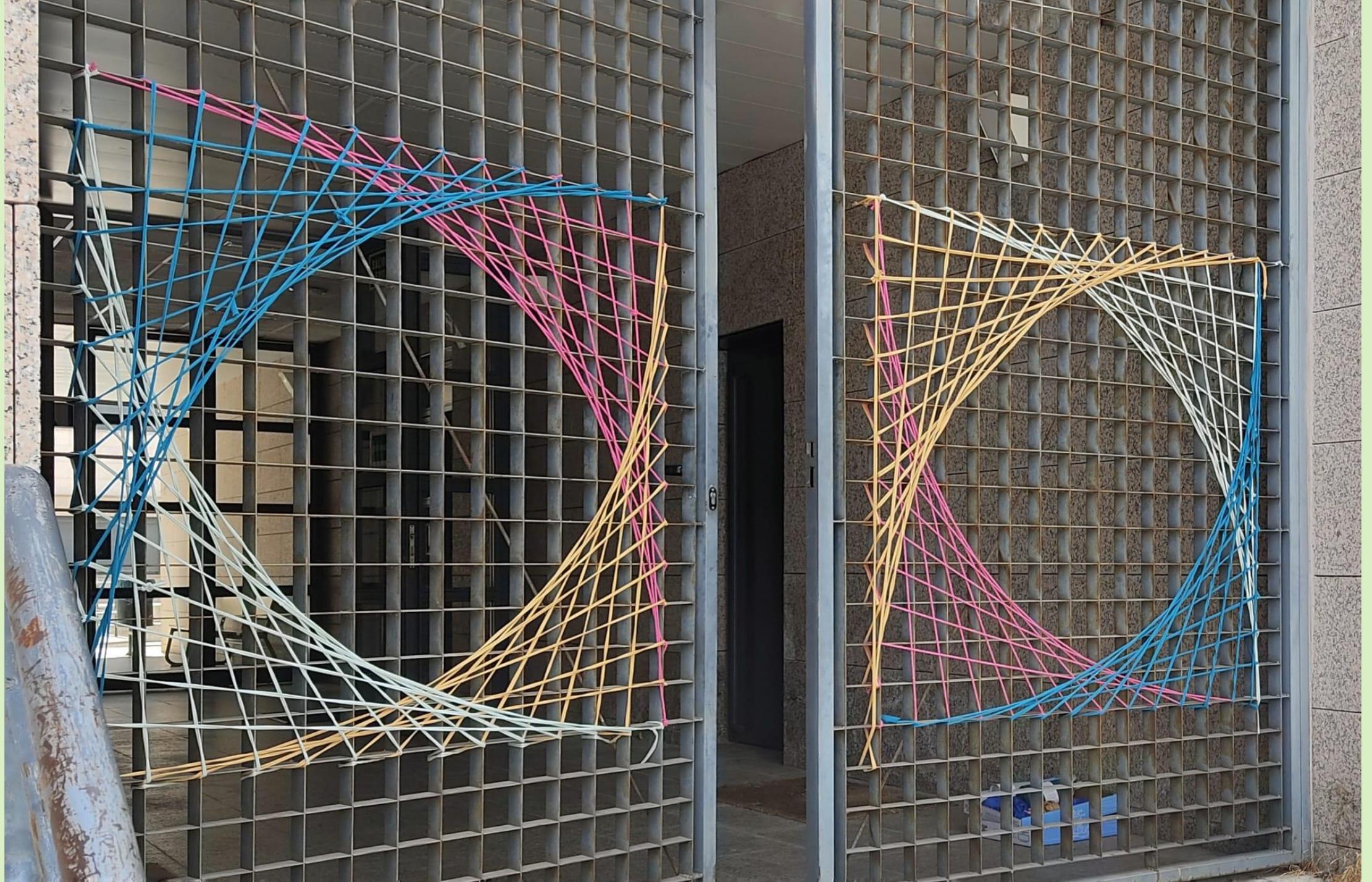
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|----------|---------------------|---------------|-------------|
| Libro JA | Hilogramas | Probabilidad | Movimientos |
| Álgebra | Geometría analítica | Trigonometría | Análisis |













Competencias específicas:

4. Utilizar los principios del pensamiento computacional organizando datos, descomponiendo en partes, reconociendo patrones, creando algoritmos para modelizar situaciones.
5. Reconocer y utilizar conexiones entre los diferentes elementos matemáticos interconectando conceptos y procedimientos.
6. Identificar las matemáticas implicadas en situaciones reales susceptibles de ser abordadas en términos matemáticos.
9. Desarrollar destrezas personales, identificando y gestionando emociones, poniendo en práctica estrategias de aceptación del error como parte del proceso de aprendizaje y disfrutar aprendiendo matemáticas.
10. Desarrollar destrezas sociales reconociendo y respetando las emociones y experiencias de los demás.

Saberes básicos:

A. Sentido numérico.

Razones y proporciones: comprensión y representación de relaciones cuantitativas.

C. Sentido espacial

Construcción de figuras geométricas con herramientas manipulativas y digitales (programas de geometría dinámica y realidad aumentada, entre otros).

Transformaciones elementales como giros, traslaciones y simetrías en situaciones diversas utilizando herramientas tecnológicas o manipulativas

Modelización geométrica: resolución de problemas relacionados con el resto de sentidos matemáticos.

D. Sentido algebraico.

Obtención, mediante observación, de pautas y regularidades sencillas.

Variable: comprensión del concepto.

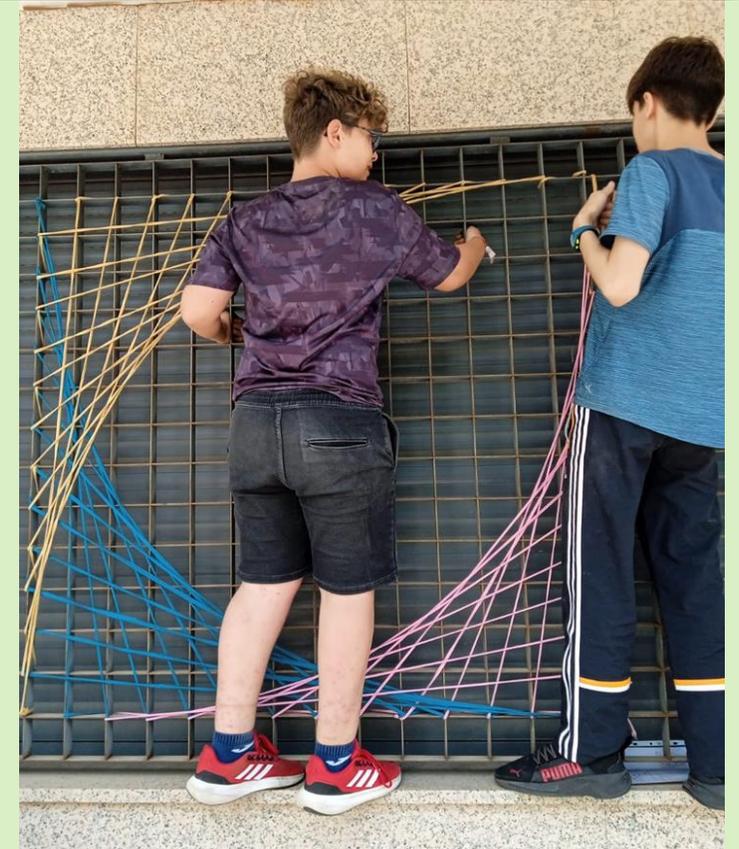
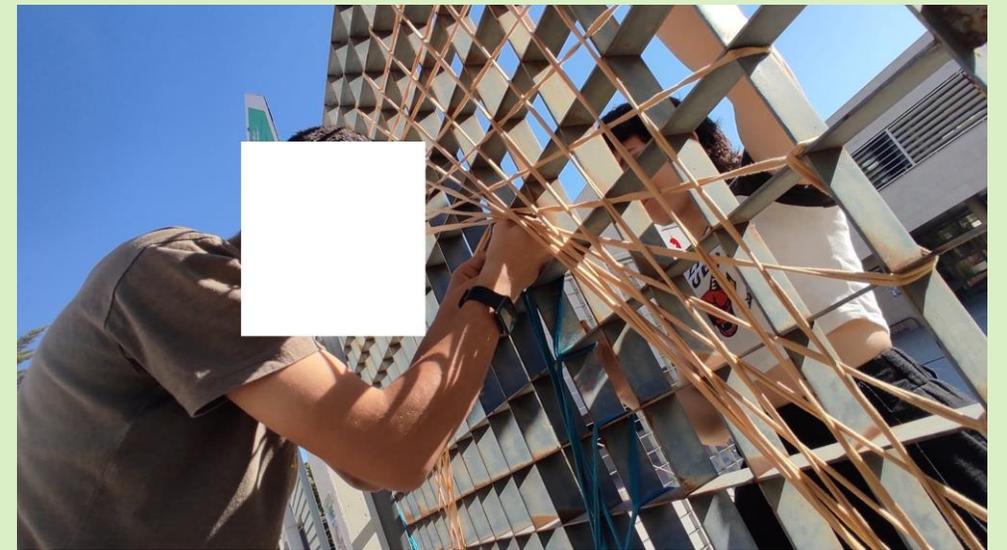
F. Sentido socioafectivo.

Estrategias de fomento de la curiosidad, la iniciativa, la perseverancia y la resiliencia en el aprendizaje de las matemáticas.

Estrategias de fomento de la flexibilidad cognitiva: apertura a cambios de estrategia y transformación del error en oportunidad de aprendizaje.

Técnicas cooperativas para optimizar el trabajo en equipo y compartir y construir conocimiento matemático.

Conductas empáticas y estrategias de gestión de conflictos.



¿Qué vamos a aprender a hacer?

Archivo Edita Vista Opciones Herramientas Ventana Ayuda

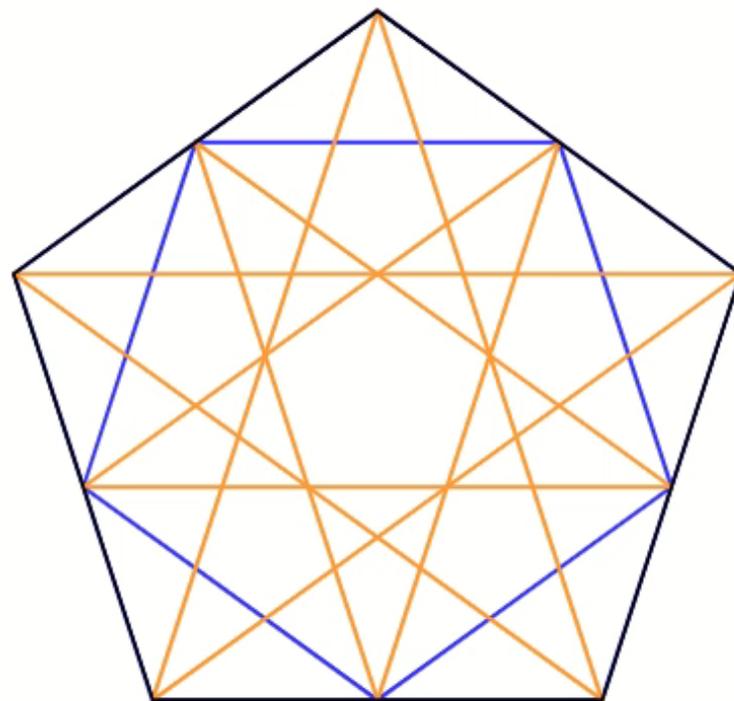
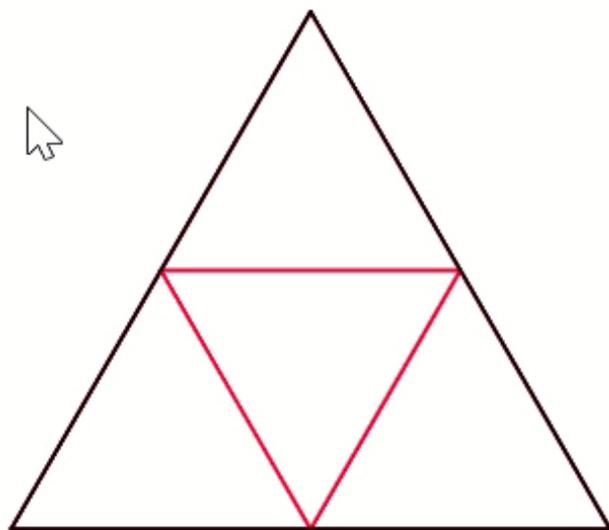
Entrada:

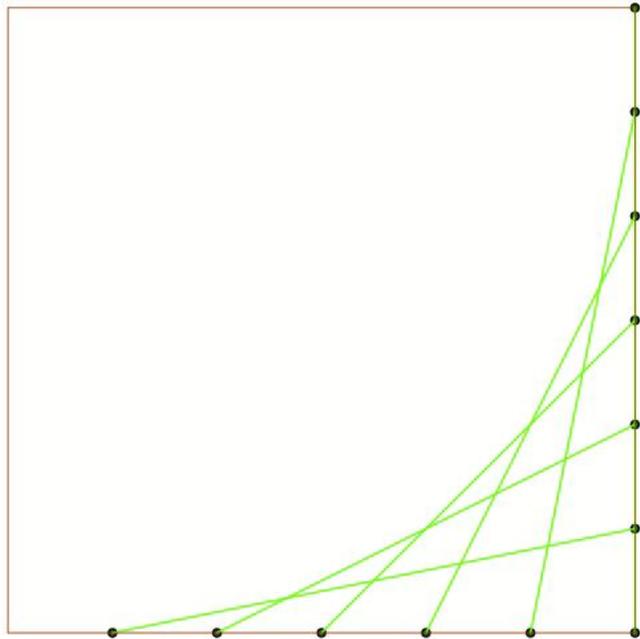
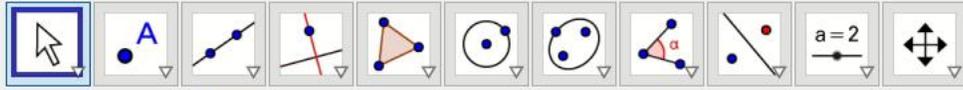
$n = 2$

$j = 8$

The image shows a screenshot of a geometry software application. The interface includes a menu bar with options: Archivo, Edita, Vista, Opciones, Herramientas, Ventana, and Ayuda. Below the menu is a toolbar with various geometric tools such as a selection tool, text tool (labeled 'A'), line tool, perpendicular line tool, angle tool, circle tool, ellipse tool, arc tool, and a tool labeled 'a=2'. There are also undo and redo buttons, and a help button. Below the toolbar is an input field labeled 'Entrada:'. The main workspace contains three geometric figures: a square with a diamond (rhombus) inscribed inside it, a circle, and two sliders. The top slider is labeled 'n = 2' and the bottom slider is labeled 'j = 8'. The sliders are represented by a horizontal line with a circular knob.

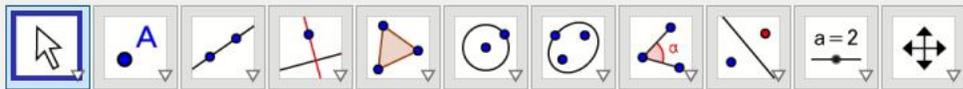
$n = 2$



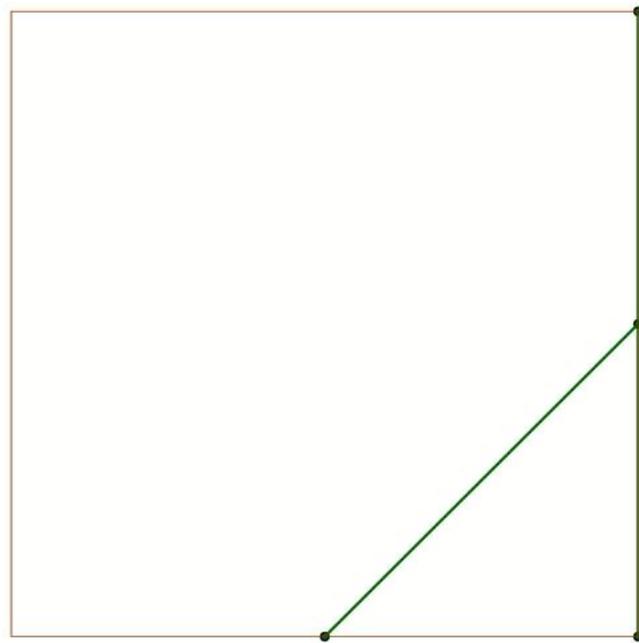


¿Todos los segmentos tienen la misma longitud?

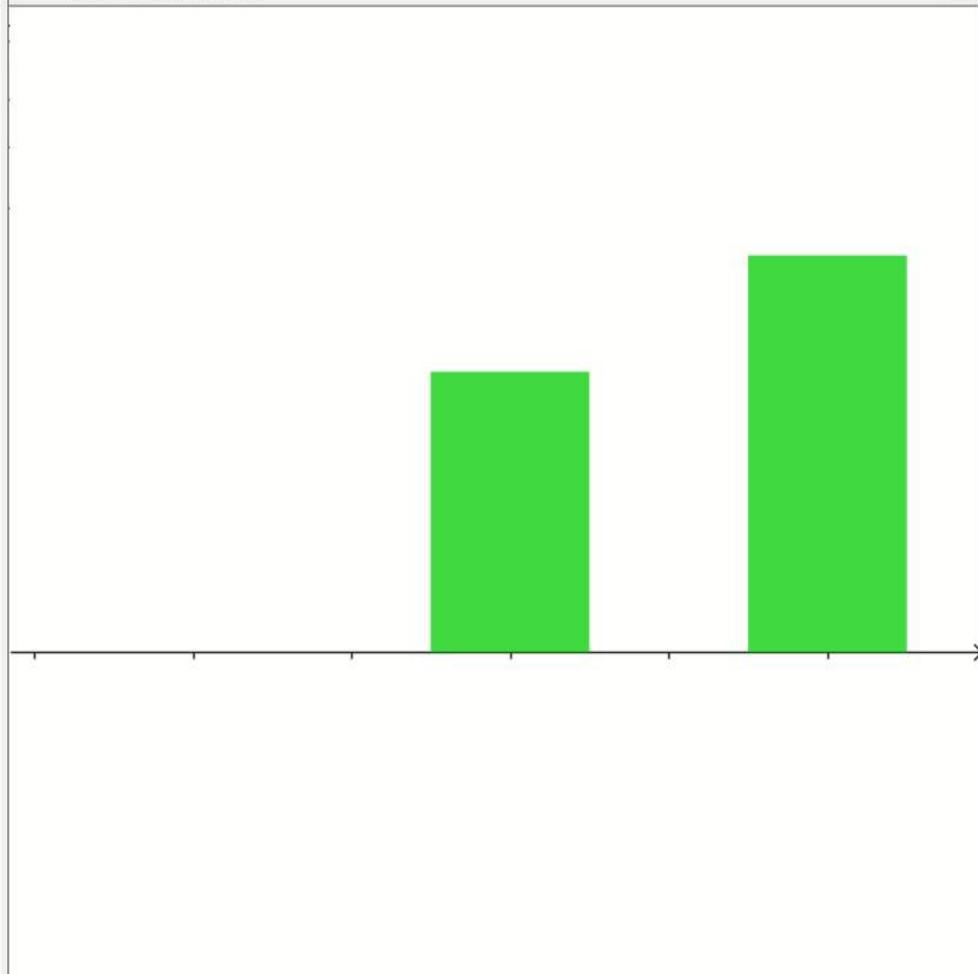




▶ Vista Gráfica

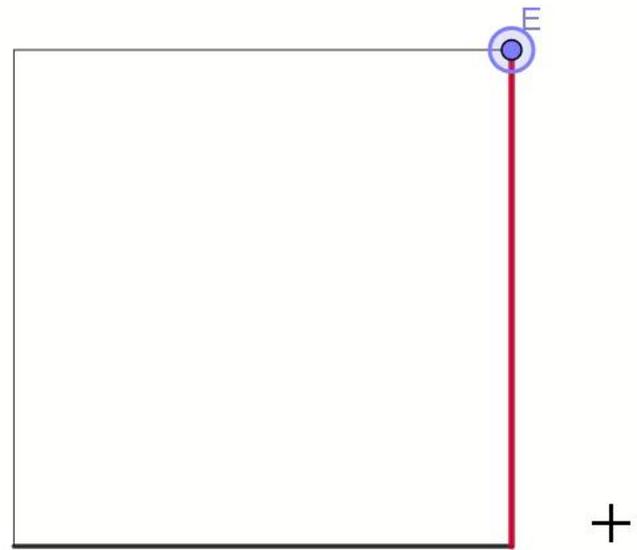
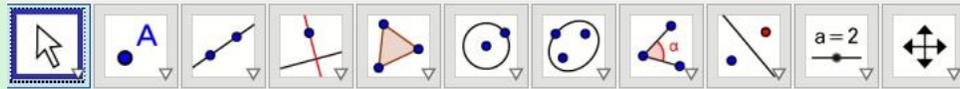


▶ Vista Gráfica 2



Entrada:

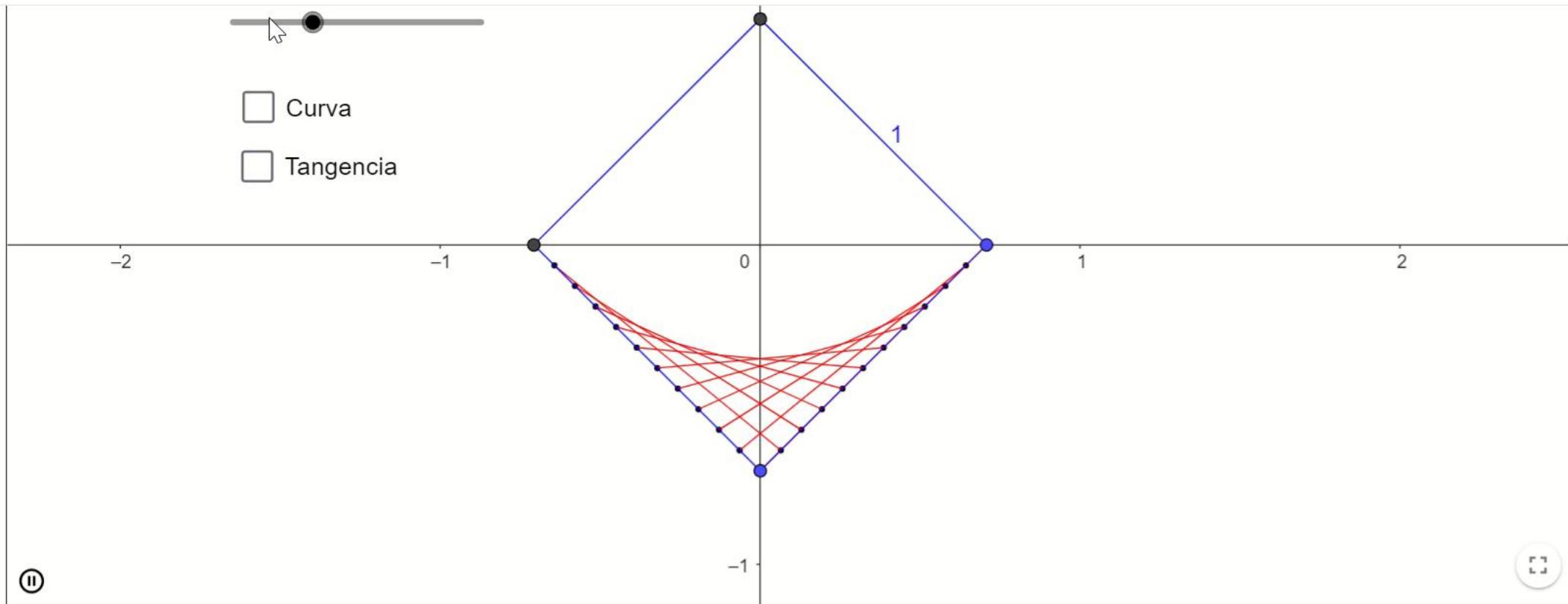






Curva

Tangencia



Dividimos un segmento en dos partes iguales (punto medio):

The screenshot shows the GeoGebra Classic 5 interface. The window title is "GeoGebra Classic 5". The menu bar includes "Archivo", "Edita", "Vista", "Opciones", "Herramientas", "Ventana", and "Ayuda". The toolbar contains various geometric tools, with the "Segmento" tool highlighted. Below the toolbar is an "Entrada:" text box. The interface is split into two views: "Vista Algebraica" on the left and "Vista Gráfica" on the right. In the algebraic view, two points are listed: $A = (-0.24, 2.26)$ and $B = (6.28, -0.38)$. In the graphic view, two blue dots representing points A and B are plotted on a coordinate plane. A mouse cursor is visible in the bottom-left corner of the graphic view.

Dividimos un segmento en tres partes iguales:

GeoGebra Classic 5 (2)

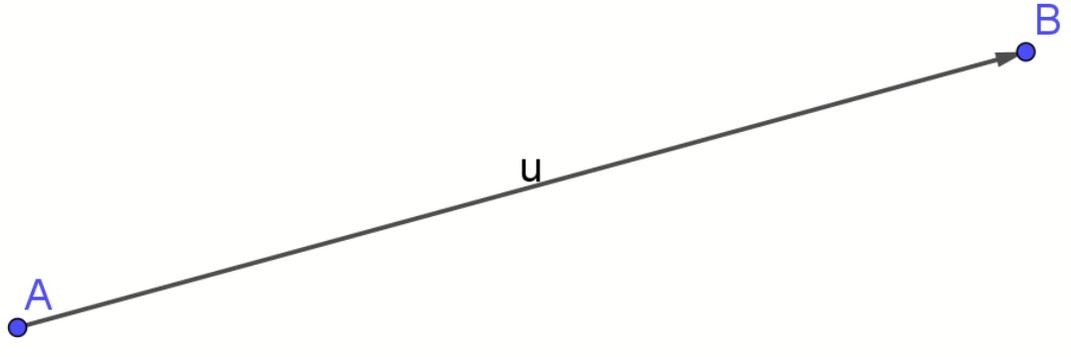
Archivo Edita Vista Opciones Herramientas Ventana Ayuda

Entrada: I

Vista Algebraica

- $A = (-1.9, 0.68)$
- $B = (9.4, 3.78)$
- $u = \begin{pmatrix} 11.3 \\ 3.1 \end{pmatrix}$

Vista Gráfica



The image shows the GeoGebra Classic 5 interface. The main workspace displays a vector u starting at point A and ending at point B . The vector is represented by a black arrow with a blue dot at each end. The points A and B are also marked with blue dots. The vector u is labeled with the letter 'u' in the middle. The algebraic view on the left shows the coordinates of A and B , and the vector u as a column vector.

Dividimos un segmento en cinco partes iguales “a mano”:

GeoGebra Classic 5

Archivo Editar Vista Opciones Herramientas Ventana Ayuda

Entrada: `Traslada(A, Vector(u / 5))`

► Vista Algebraica ► Vista Gráfica

- $A = (-0.44, -1.02)$
- $B = (10.18, 0.8)$
- $\mathbf{u} = \begin{pmatrix} 10.62 \\ 1.82 \end{pmatrix}$
- $A' = (1.68, -0.66)$

The image shows a screenshot of the GeoGebra Classic 5 software interface. The main workspace displays a coordinate plane with three points: A, A', and B. A line segment connects point A to point B. Point A' is located on the segment AB, closer to A. A vector labeled 'u' is shown as an arrow pointing from A to B. The algebraic view on the left lists the coordinates for points A, B, and A', and the vector u. The input field at the top shows the command 'Traslada(A, Vector(u / 5))', which is used to translate point A by one-fifth of the vector u to find point A'.

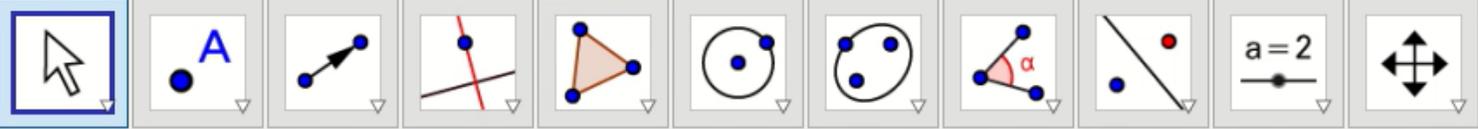
Dividimos un segmento en cinco partes iguales. Comando **Secuencia**:

The screenshot shows the GeoGebra Classic 5 interface. The title bar reads "GeoGebra Classic 5". The menu bar includes "Archivo", "Edita", "Vista", "Opciones", "Herramientas", "Ventana", and "Ayuda". The toolbar contains various geometric tools, with the "Secuencia" tool (represented by a blue square with a white arrow) highlighted. Below the toolbar is an "Entrada:" field. The main workspace is split into two views: "Vista Algebraica" (Algebraic View) on the left and "Vista Gráfica" (Graphic View) on the right. In the Algebraic View, the "Punto" (Point) section shows two points: $A = (-2.2, 3.56)$ and $B = (6.68, 0.04)$. The "Vector" section shows a vector $u = \begin{pmatrix} 8.88 \\ -3.52 \end{pmatrix}$. In the Graphic View, a vector u is drawn from point A to point B .

Empezamos con un hilorama triangular de 10 hebras:

GeoGebra Classic 5

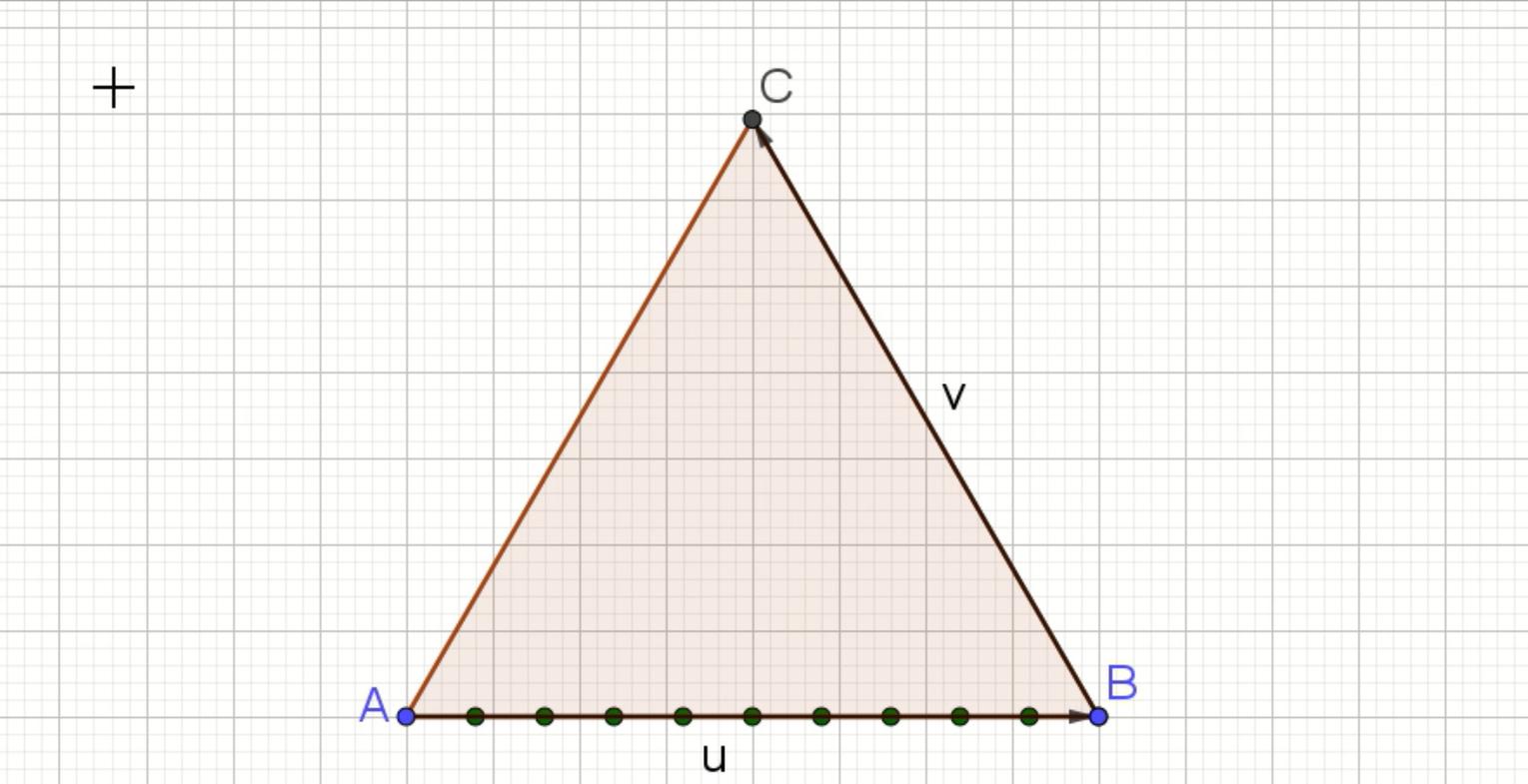
Archivo Edita Vista Opciones Herramientas Ventana Ayuda



Entrada: `Secuencia(Traslada(A, Vector(k u / 10)), k, 1, 10)`

► Vista Algebraica ► Vista Gráfica

- $A = (0, -2)$
- $B = (8, -2)$
- $f = 8$
- $\text{polígono1} = 27$
- $u = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$
- $v = \begin{pmatrix} -4 \\ 6.93 \end{pmatrix}$



Dividimos un segmento en un número variable n de partes iguales:

The screenshot shows the GeoGebra Classic 5 interface. The top menu bar includes "Archivo", "Edita", "Vista", "Opciones", "Herramientas", "Ventana", and "Ayuda". The toolbar contains various geometric tools, with the "a=2" tool highlighted. The "Entrada:" field is empty. The left sidebar shows the "Vista Algebraica" (Algebraic View) with the following data:

- Punto**
 - $A = (-2.2, 3.56)$
 - $B = (6.68, 0.04)$
- Vector**
 - $u = \begin{pmatrix} 8.88 \\ -3.52 \end{pmatrix}$

The "Vista Gráfica" (Graphic View) shows a coordinate plane with two points, A and B, connected by a line segment. The segment is labeled with the vector u . The vector u is represented by an arrow pointing from point A to point B.

Dinamizamos el hilorama

GeoGebra Classic 5 (2)

Archivo Edita Vista Opciones Herramientas Ventana Ayuda

Entrada:

Vista Algebraica Vista Gráfica

Lista

- $I1 = \{(-2.5, -1)$
- $I2 = \{(1.75, -0)$
- $I3 = \{4.27, 3.1$
- $I3' = \{4.27, 3.1$
- $I3'_1 = \{4.27, 3.1$

Número

- $n = 27$

Polígono

- polígono1 =

Punto

- $A = (-3, -1)$
- $B = (2, -1)$

Seguimos con el cuadrado

GeoGebra Classic 5 (2)

Archivo Edita Vista Opciones Herramientas Ventana Ayuda

Entrada: I

Vista Algebraica Vista Gráfica

Lista

- I1 = {(-2.44, -
- I2 = {(2, -0.44
- I3 = {4.48, 4.0

Número

- n = 9

Polígono

- polígono1 =

Punto

- A = (-3, -1)
- B = (2, -1)
- D = (-0.5, 1.5)

Segmento

The image shows a GeoGebra workspace with a grid. A square ABCD is drawn with vertices A(-3, -1), B(2, -1), C(2, 1), and D(-0.5, 1.5). A red curve is plotted, starting from point A and ending at point C, passing through several points on the bottom and right sides of the square. A slider on the right is set to n=9. The algebraic view on the left shows the coordinates of the points and the value of n.

Nos surge un problema

GeoGebra Classic 5 (2)

Archivo Edita Vista Opciones Herramientas Ventana Ayuda

Entrada: I

Vista Algebra Vista Gráfica

Lista

- $I1 = \{(-2.62, -$
- $I2 = \{(2, 3.62$

Número

- $n = 13$

Polígono

- polígono1 =

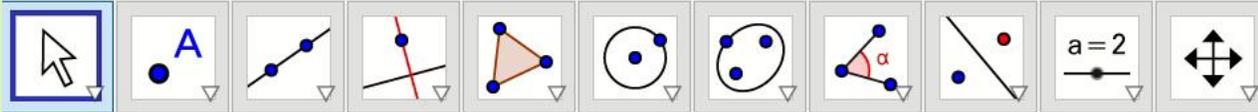
Punto

- $A = (-3, -1)$
- $B = (2, -1)$
- $D = (-0.5, 1.5)$

Segmento

- $f = 5$

The diagram shows a square with vertices A, B, C, and E. Point D is inside the square. The bottom side AB and the right side BC are filled with a grid of green points. The horizontal distance between A and B is labeled 'u', and the vertical distance between B and C is labeled 'v'. A slider on the right is set to 'n = 13'.



Entrada:

Vista Algebra

Lista

- I1 = {(-2.62, -
- I2 = {(2, 3.62
- I3 = {6.53, 5.

Número Lista I3

• n = 13

Polígono

• polígono1 =

Punto

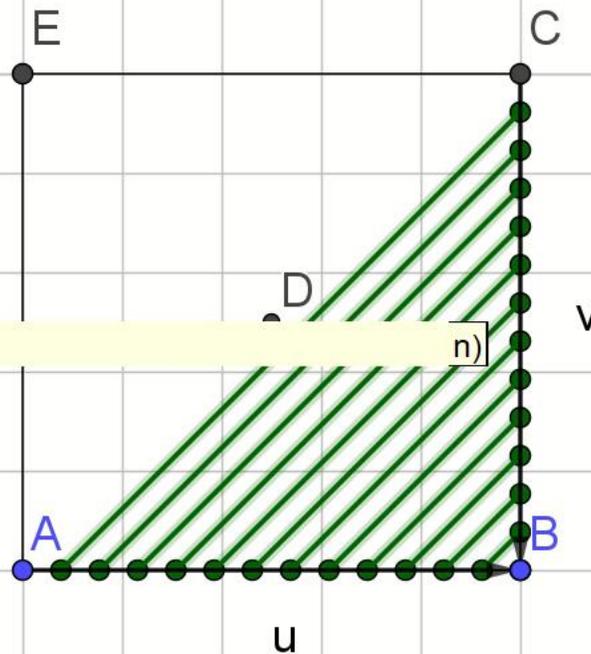
• A = (-3, -1)

• B = (2, -1)

• D = (-0.5, 1.5

Segmento

Vista Gráfica



n = 13

Y ahora uno compuesto

GeoGebra Classic 5 (2)

Archivo Editar Vista Opciones Herramientas Ventana Ayuda

Entrada: I

Vista Algebraica Vista Gráfica

Lista

- I1 = {(-2.62, -
- I2 = {(2.12, -
- I3 = {4.75, 4.
- I4 = {(3.23, 3
- m1 = {{4.75,

Número

- n = 13

Polígono

- polígono1 =

Punto

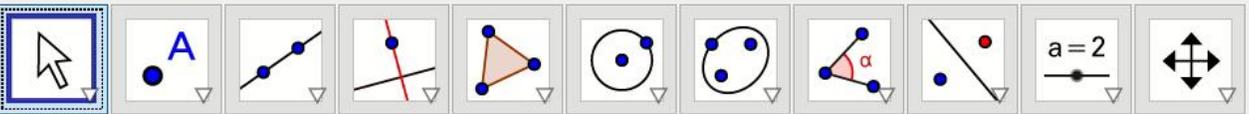
- A = (-3, -1)
- B = (2, -1)

The image shows a screenshot of the GeoGebra Classic 5 software interface. The main workspace is a coordinate grid with a complex geometric construction. A set of points A, B, C, D, E, F are visible. A red mesh of lines connects these points, forming a complex, multi-lobed shape. A slider on the right is labeled 'n = 13'. The left sidebar shows the 'Lista' (List) of objects, including several intervals (I1, I2, I3, I4), a number (n = 13), a polygon (polígono1), and points (A, B). The top menu bar includes 'Archivo', 'Editar', 'Vista', 'Opciones', 'Herramientas', 'Ventana', and 'Ayuda'. The toolbar contains various geometric tools like point, line, circle, and polygon.

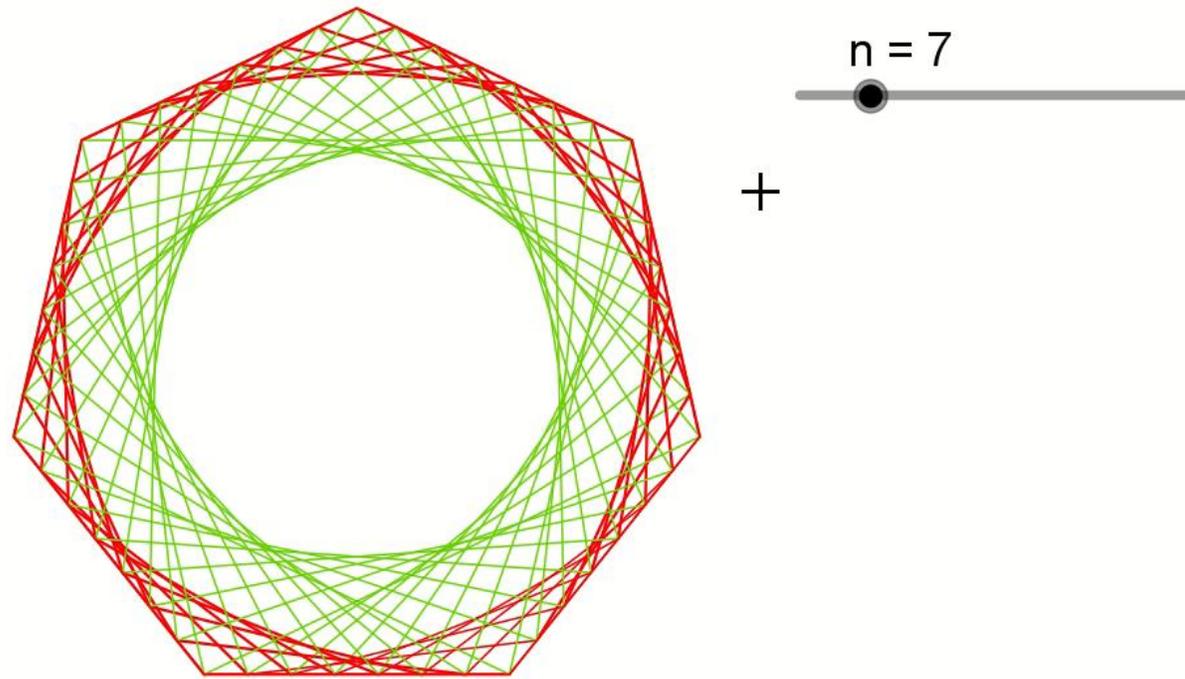
Y más

GeoGebra Classic 5 (2)

Archivo Edita Vista Opciones Herramientas Ventana Ayuda



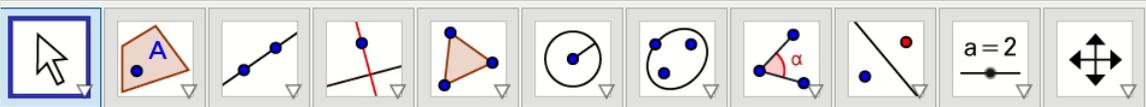
Entrada:



Vamos con el círculo

GeoGebra Classic 5

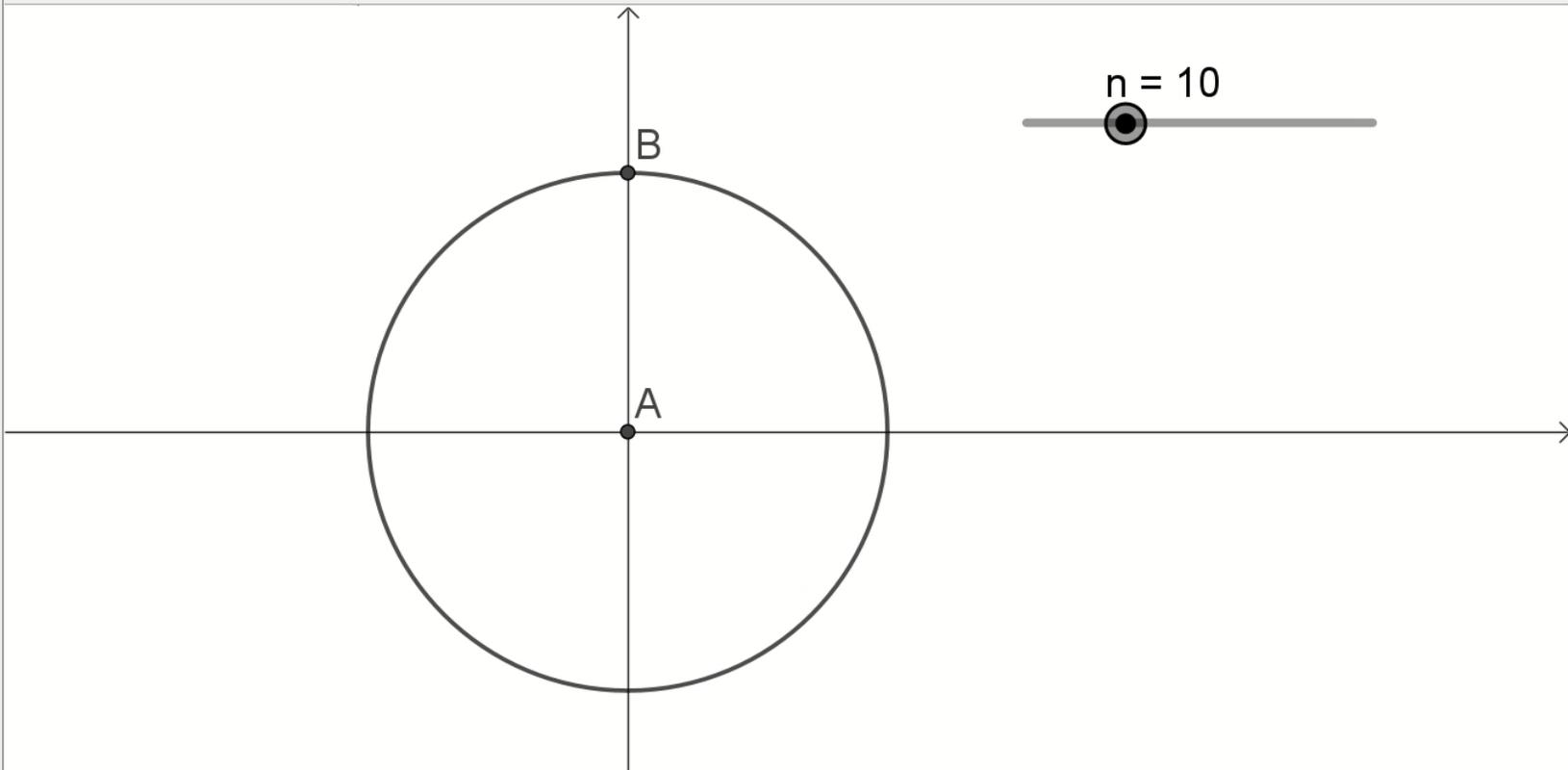
Archivo Edita Vista Opciones Herramientas Ventana Ayuda



Entrada: Secuencia(<Expresión>, <Variable>, <Valor inicial>, <Valor final>)

Vista Algebraica X Vista Gráfica X

- Cónica
 - $c: x^2 + y^2 = 9$
- Número
 - $n = 10$
- Punto
 - $A = (0, 0)$
 - $B = (0, 3)$



The image shows the GeoGebra Classic 5 interface. The main workspace displays a Cartesian coordinate system with a circle centered at point A(0,0) and passing through point B(0,3). The circle's equation is $x^2 + y^2 = 9$. A slider for the variable n is set to 10. The left sidebar shows the algebraic view with the following objects listed: a conic section $c: x^2 + y^2 = 9$, a number $n = 10$, and two points $A = (0, 0)$ and $B = (0, 3)$. The top menu bar includes Archivo, Edita, Vista, Opciones, Herramientas, Ventana, and Ayuda. The toolbar contains various geometric tools, and the input field shows the command `Secuencia(<Expresión>, <Variable>, <Valor inicial>, <Valor final>)`.

Nos surge otro problema

GeoGebra Classic 5

Archivo Edita Vista Opciones Herramientas Ventana Ayuda

Entrada: I

Vista Algebraica Vista Gráfica

Cónica
● **c: $x^2 + y^2 = 1$**

Lista
● **l1 = {(-0.25, 0.97)}**

Número
● **n = 25**

Punto
● **A = (0, 0)**
● **B = (0, 1)**
○ **B' = (-0.31, 0.95)**
○ **B'_1 = (-0.59, 0.81)**
○ **B'_2 = (-0.81, 0.59)**

The image shows the GeoGebra Classic 5 interface. The main workspace displays a coordinate system with a unit circle centered at the origin A(0,0). Point B is at (0,1). The circle is divided into 25 equal segments by green points. A slider on the right is set to n=25. The algebraic view on the left shows the circle equation $x^2 + y^2 = 1$ and a list of points, including the current point B' and its neighbors B'_1 and B'_2.

Pues seguimos con el problema

GeoGebra Classic 5

Archivo Edita Vista Opciones Herramientas Ventana Ayuda

Entrada:

Vista Algebraic Vista Gráfica

$c: x^2 + y^2 = 1$

Lista

- $I1 = \{(-0.46, 0.98)\}$
- $I2 = \{1.33, 1.33\}$

Número

- $n = 13$

Punto

- $A = (0, 0)$
- $B = (0, 1)$
- $B' = (-0.31, 0.95)$
- $B'_1 = (-0.59, 0.81)$
- $B'_2 = (-0.81, 0.59)$
- $B'_3 = (-0.95, 0.31)$
- $B'_4 = (-1, 0)$
- $B'_5 = (-0.95, -0.31)$
- $B'_6 = (-0.81, -0.59)$
- $B'_7 = (-0.59, -0.81)$
- $B'_8 = (-0.31, -0.95)$
- $B'_9 = (0, -1)$
- $B'_{10} = (0.31, -0.95)$
- $B'_{11} = (0.59, -0.81)$
- $B'_{12} = (0.81, -0.59)$
- $B'_{13} = (0.95, -0.31)$
- $B'_{14} = (1, 0)$
- $B'_{15} = (0.95, 0.31)$
- $B'_{16} = (0.81, 0.59)$
- $B'_{17} = (0.59, 0.81)$
- $B'_{18} = (0.31, 0.95)$
- $B'_{19} = (0, 1)$

The image shows the GeoGebra Classic 5 interface. The main workspace displays a coordinate system with a unit circle centered at the origin A(0,0). Point B(0,1) is marked at the top of the circle. A complex green polygonal path is drawn on the circle, connecting 13 points. To the right of the workspace, a slider is labeled 'n = 13'. The left sidebar shows the 'Vista Algebraic' with a list of objects: a circle 'c' with equation $x^2 + y^2 = 1$, a list containing two points 'I1' and 'I2', and a number 'n = 13'. The 'Vista Gráfica' shows the geometric construction.

¿Y si probamos otra cosa?

GeoGebra Classic 5

Archivo Edita Vista Opciones Herramientas Ventana Ayuda

Entrada:

Vista Algebraic Vista Gráfica

Lista I2

Secuencia(Segmento(), k, 1, n - 1)

Propiedades... OK Cancela Aplicar

$c: x^2 + y^2 = 1$

$I1 = \{(-0.46, 0.98), (-0.92, 0.39), (-0.92, -0.39), (-0.46, -0.98), (0, -1)\}$

$I2 = \{1.33, 1.33, 1.33, 1.33, 1.33\}$

$n = 13$

Punto

$A = (0, 0)$

$B = (0, 1)$

$B' = (-0.31, 0.95)$

$B'_1 = (-0.59, 0.81)$

$B'_2 = (-0.81, 0.59)$

$B'_3 = (-0.95, 0.31)$

$B'_4 = (-0.98, 0.17)$

$B'_5 = (-0.98, -0.17)$

$B'_6 = (-0.95, -0.31)$

$B'_7 = (-0.81, -0.59)$

$B'_8 = (-0.59, -0.81)$

$B'_9 = (-0.31, -0.95)$

$B'_{10} = (0, -0.98)$

$B'_{11} = (0.17, -0.98)$

$B'_{12} = (0.31, -0.95)$

$B'_{13} = (0.59, -0.81)$

$B'_{14} = (0.81, -0.59)$

$B'_{15} = (0.95, -0.31)$

$B'_{16} = (0.98, -0.17)$

$B'_{17} = (0.98, 0.17)$

$B'_{18} = (0.95, 0.31)$

$B'_{19} = (0.81, 0.59)$

$B'_{20} = (0.59, 0.81)$

$B'_{21} = (0.31, 0.95)$

$B'_{22} = (0.17, 0.98)$

$B'_{23} = (0, 0.98)$

$B'_{24} = (-0.17, 0.98)$

$B'_{25} = (-0.31, 0.95)$

$B'_{26} = (-0.59, 0.81)$

$B'_{27} = (-0.81, 0.59)$

$B'_{28} = (-0.95, 0.31)$

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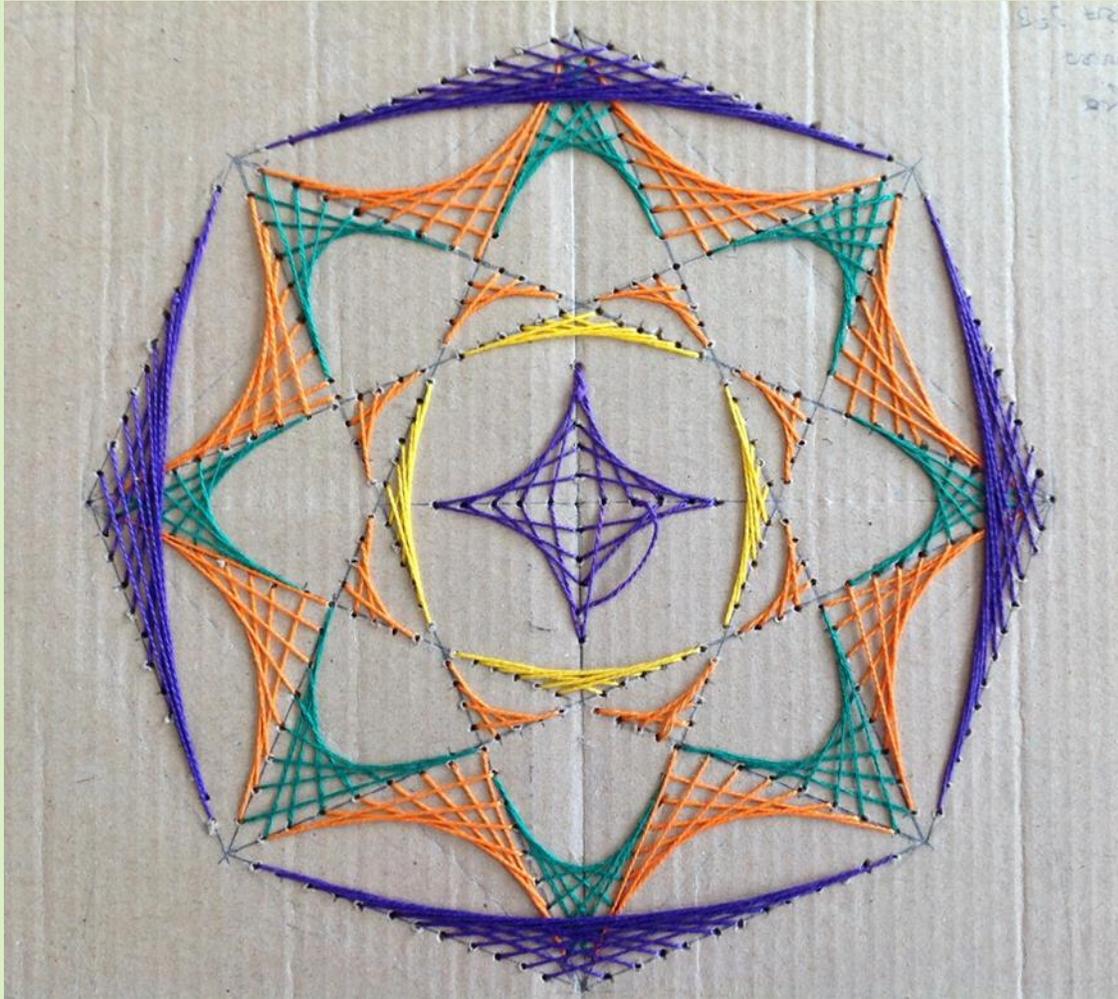
Vamos a complicarlo un poco

The image shows a screenshot of the GeoGebra software interface. The main workspace displays a circle with a point A at the origin $(0, 0)$ and a point B at $(0, 1)$. A complex geometric construction is shown, consisting of a series of green and red line segments forming a path along the circle's circumference. The green path is defined by the formula $\text{Secuencia}(\text{Segmento}(\text{Rota}(B, k * 2\pi / n), \text{Rota}(B, 2\pi / n (k + m))), k, 1, n)$ and the red path by $\text{Secuencia}(\text{Segmento}(\text{Rota}(B, k * 2\pi / n), \text{Rota}(B, 2\pi / n (k + r))), k, 1, n)$. The parameter r is set to -2.5 via a slider.

On the left, the algebraic view shows the following objects:

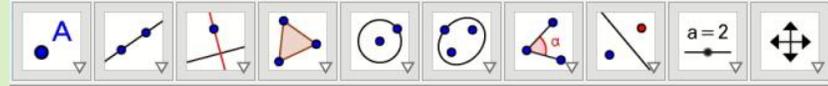
- Cónica: $c: x^2 + y^2 = 1$
- Lista:
 - $I1 = \{(-0.32, 0.95, \dots)\}$
 - $I2 = \{1.94, 1.94, \dots\}$
 - $I3 = \{0.8, 0.8, 0.1\}$
- Número:
 - $m = 8$
 - $n = 19$
 - $r = -2.5$
- Punto:
 - $A = (0, 0)$
 - $B = (0, 1)$

Two "Redefine" dialog boxes are open. The top dialog, titled "Lista I2", shows the formula for the green path. The bottom dialog, titled "Lista I3", shows the formula for the red path. Both dialogs include "Propiedades...", "OK", "Cancela", and "Aplicar" buttons.

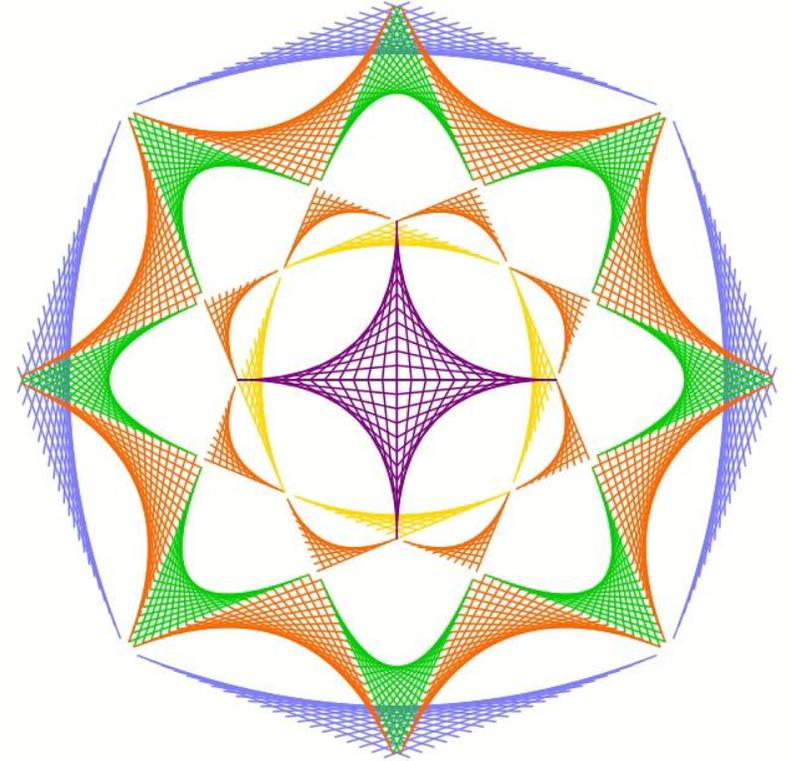


rama5.ggb

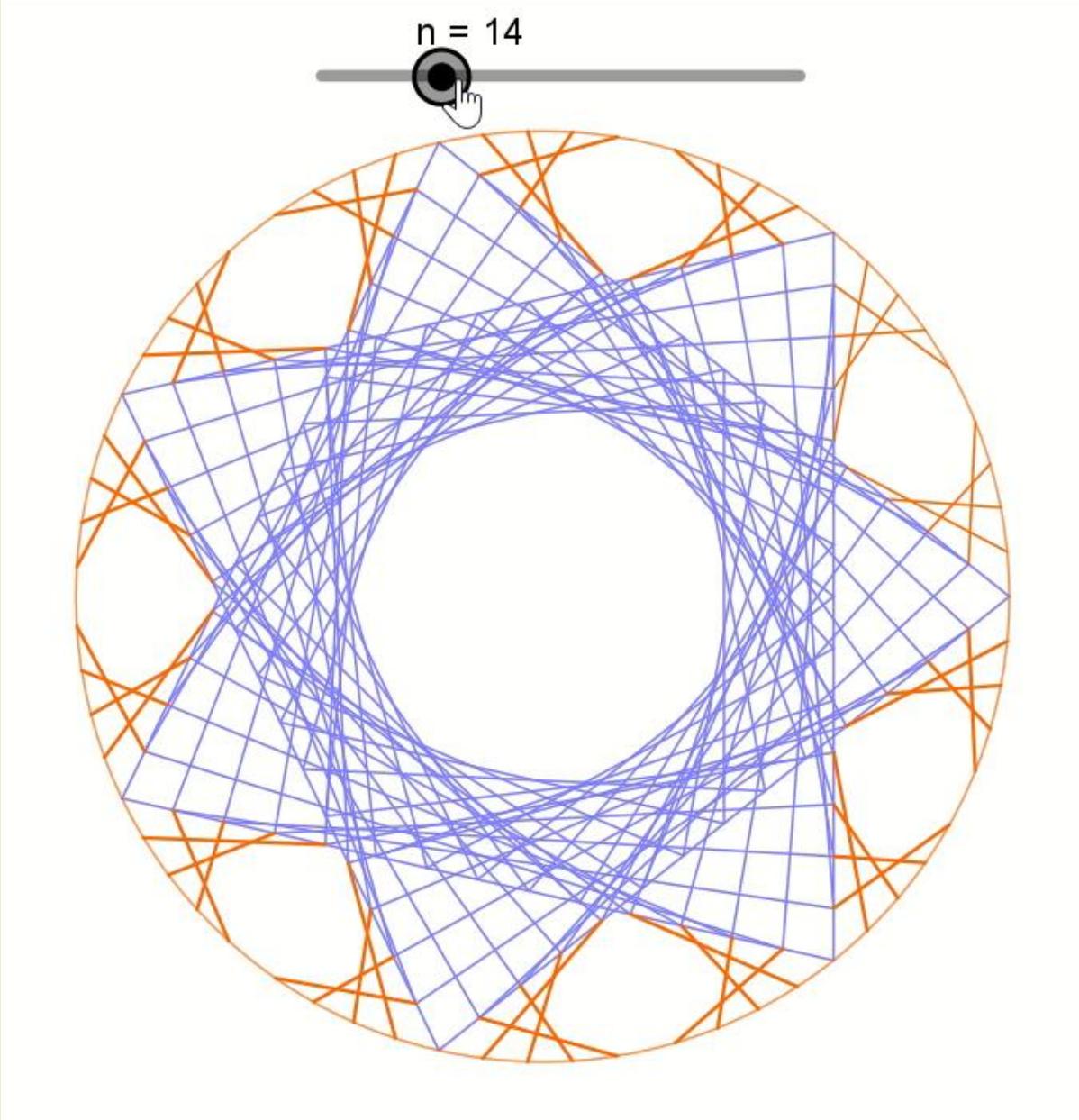
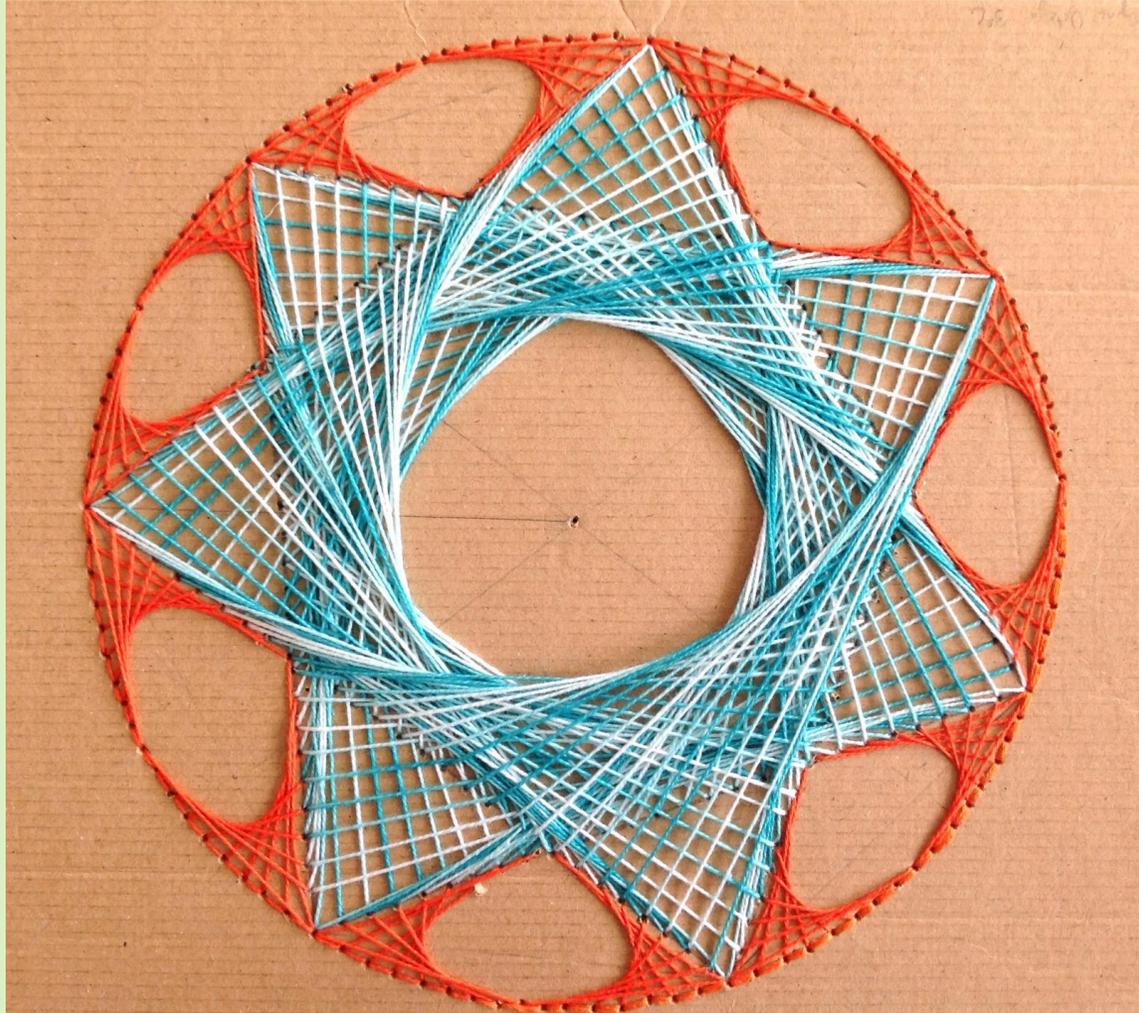
/o Edita Vista Opciones Herramientas Ventana Ayuda

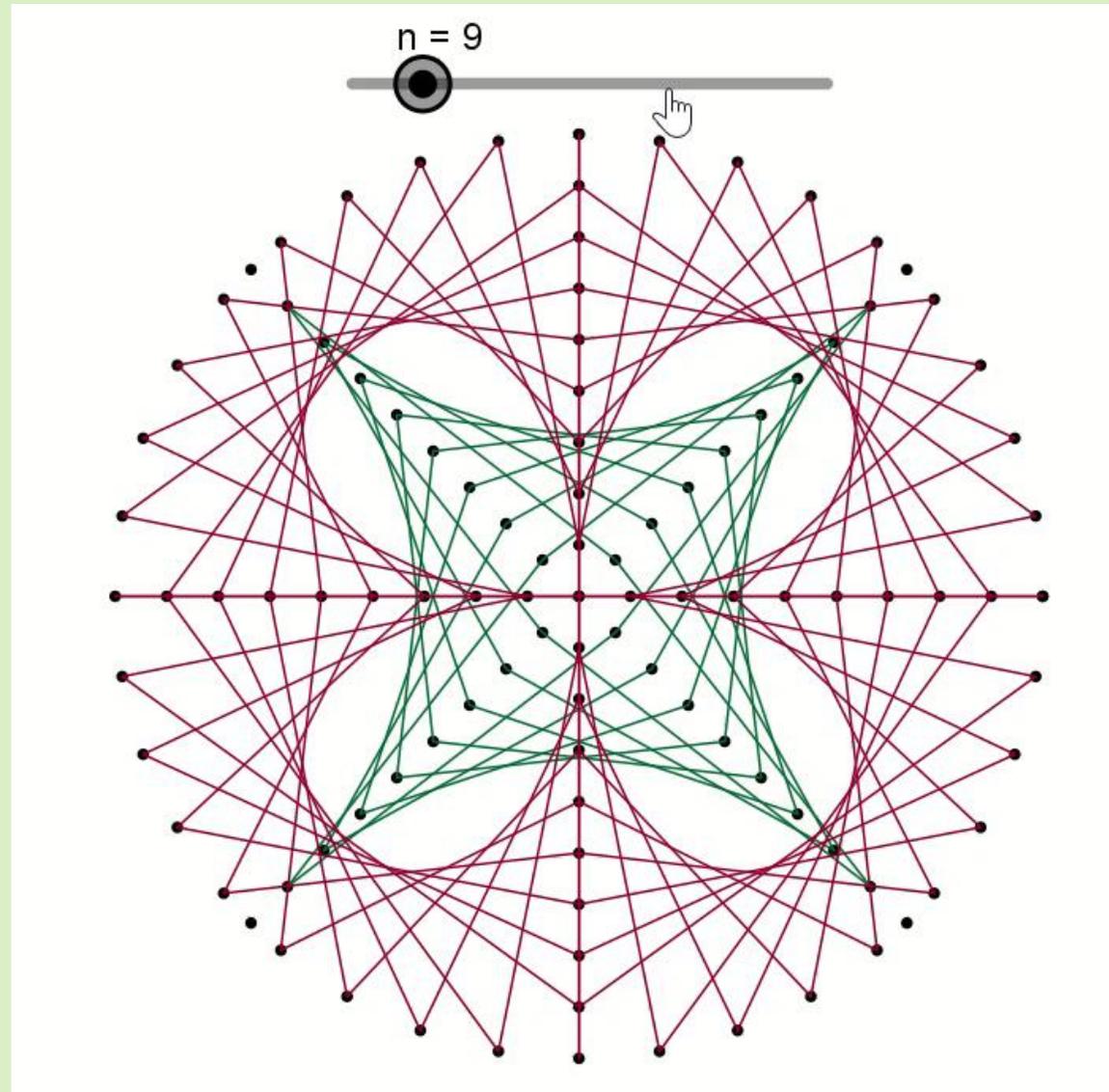
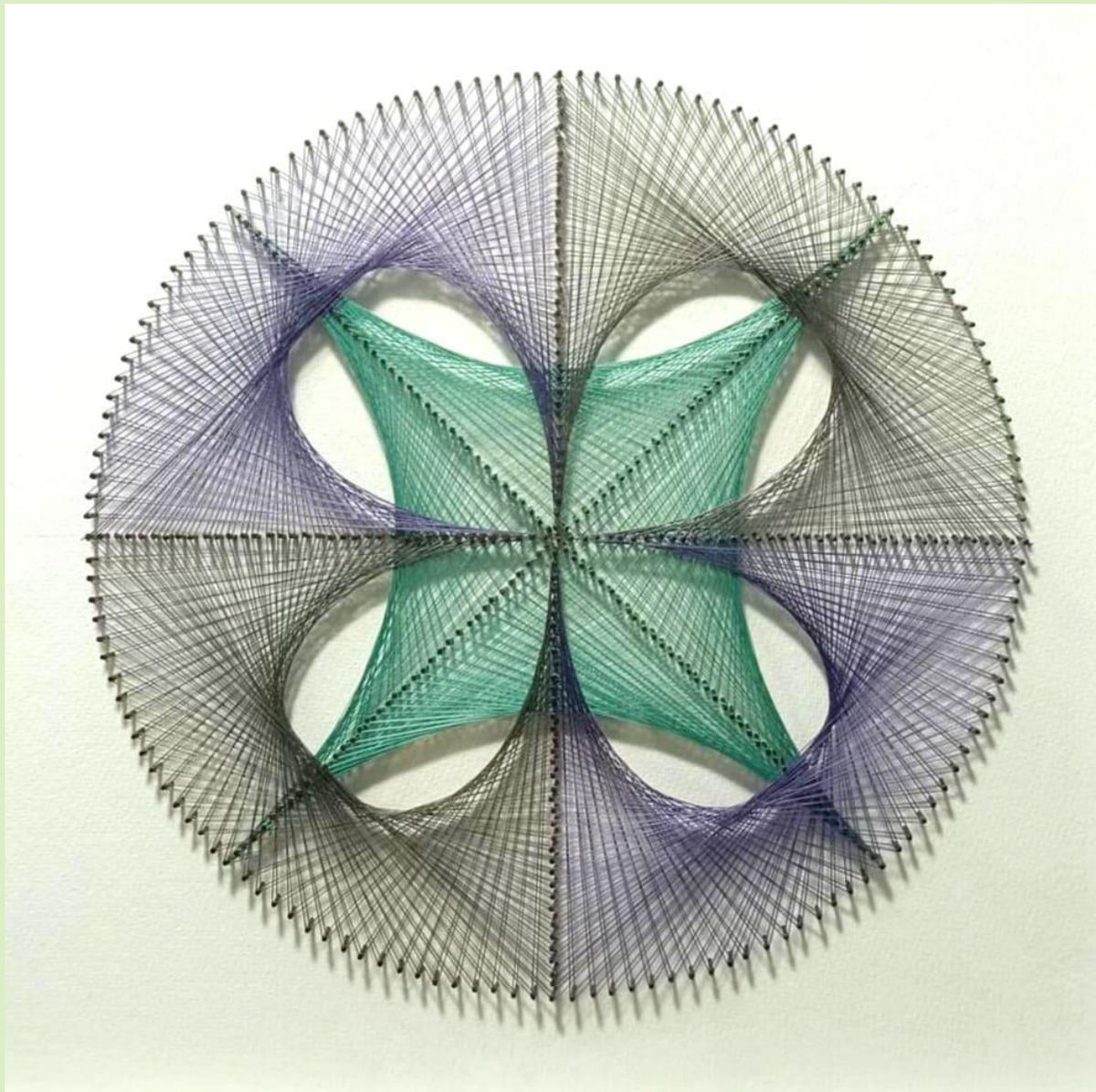


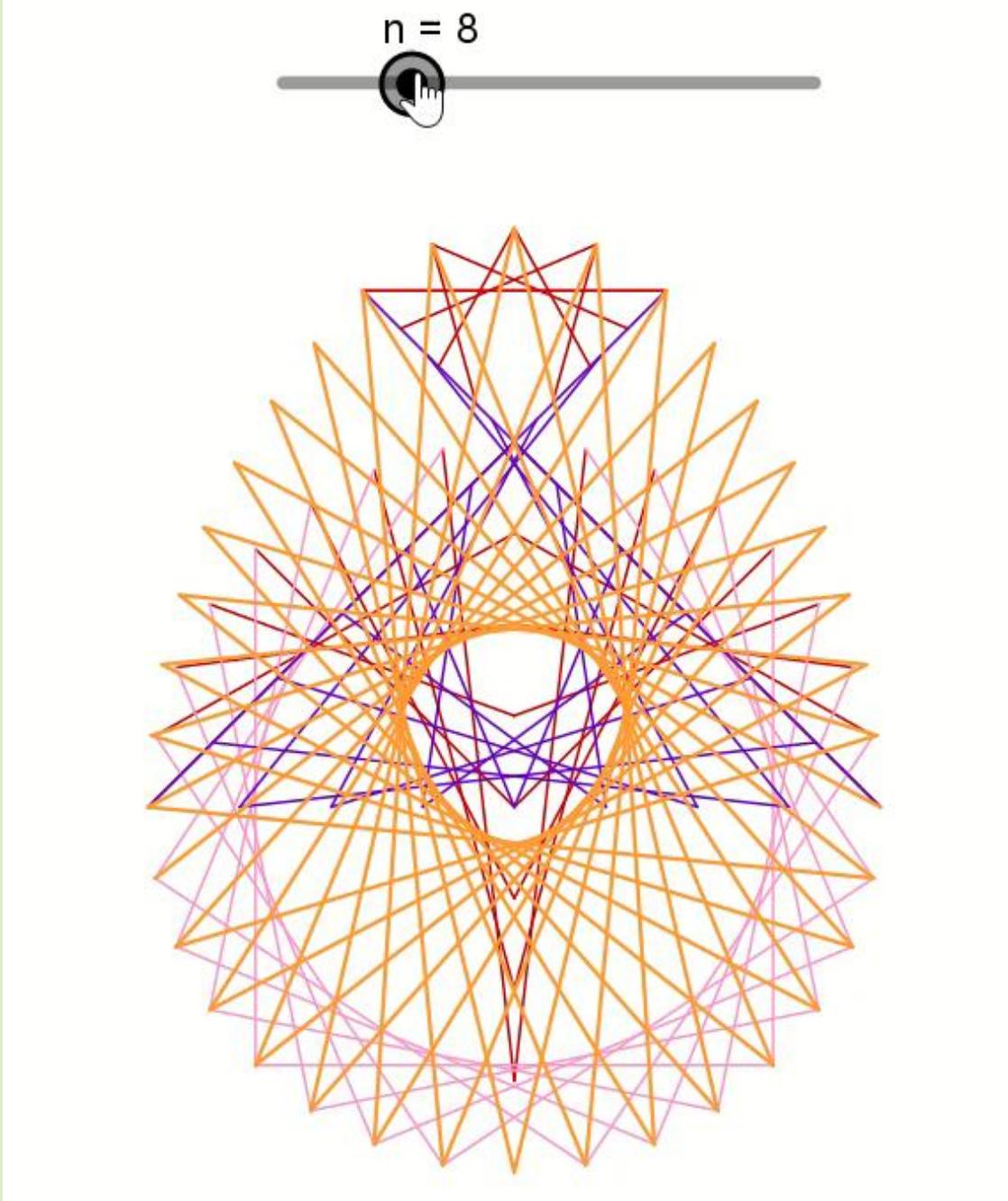
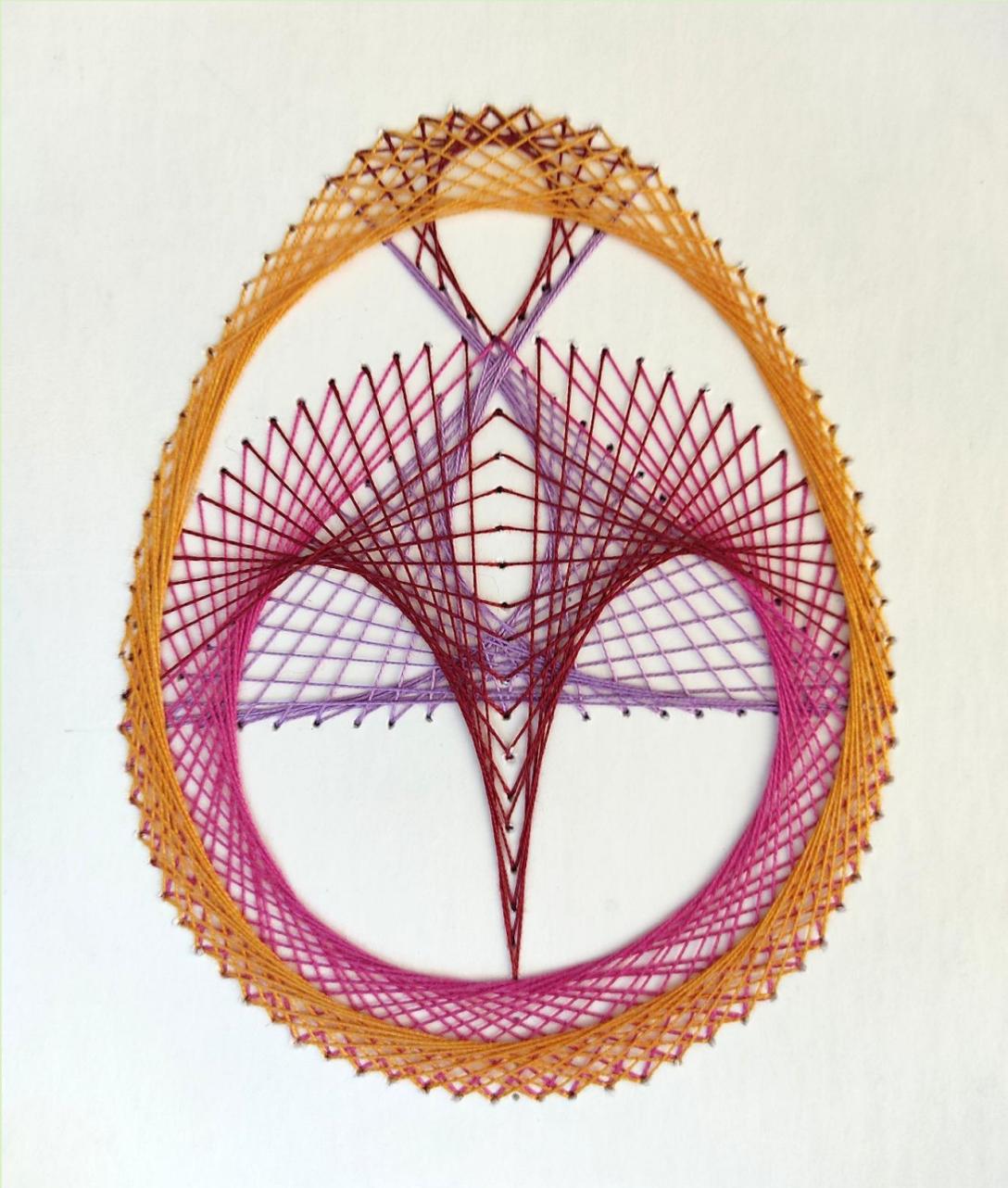
n = 22

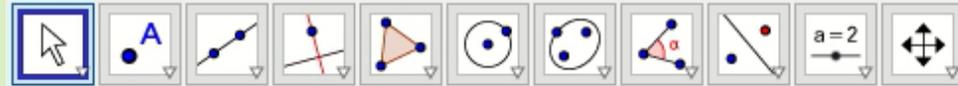


ida:









► Vista Gráfica



Jugamos con las operaciones ¿Qué pasa si multiplicamos?

GeoGebra Classic 5

Archivo Edita Vista Opciones Herramientas Ventana Ayuda

Entrada:

Vista Algebraica

- $A = (0, 0)$
- $B = (0, 5)$
- $c: x^2 + y^2 = 25$
- $n = 15$
- $m = 5$
- $I1 = \{8.66, 8.66, 8.66, 8.66, 8.66, 8.66\}$

Vista Gráfica

$n = 15$

$m = 5$

+

¿Y si dividimos?

The image shows a screenshot of the GeoGebra software interface. At the top, the menu bar includes 'Archivo', 'Edita', 'Vista', 'Opciones', 'Herramientas', 'Ventana', and 'Ayuda'. Below the menu is a toolbar with various geometric tools. The main workspace displays a circle with center point A and a point B on the circle. A sequence of green points is drawn along the lower arc of the circle, connected by a smooth curve. A 'Redefine' dialog box is open, showing the following command in the 'Lista I2' field:

```
Secuencia(Segmento(Rota(B, k * 2π / n), Rota(B, k / m 2π / n)), k, 1, n)
```

The dialog box also features a 'Propiedades...' button and 'OK', 'Cancela', and 'Aplicar' buttons. On the left side, the 'Vista Algebraica' panel is visible, showing the following objects and their coordinates:

- Cónica: $c: x^2 + y^2 = 1$
- Lista:
 - $I1 = \{(-0.17, 0.99)$
 - $I2 = \{0, 0, 0, 0, 0$
 - $I3 = \{0.82, 0.82,$
- Número:
 - $m = 1$
 - $n = 37$
 - $r = -5$
- Punto:
 - $A = (0, 0)$
 - $B = (0, 1)$

¿Y si invertimos?

The screenshot shows the GeoGebra interface with a window titled 'circular4.ggb'. The menu bar includes 'Archivo', 'Edita', 'Vista', 'Opciones', 'Herramientas', 'Ventana', and 'Ayuda'. The toolbar contains various geometric tools. The 'Entrada:' field is empty. The left sidebar shows the 'Vista Algebraica' and 'Vista Gráfica' tabs. Under 'Vista Algebraica', the following objects are listed:

- Cónica
 - $c: x^2 + y^2 = 1$
- Lista
 - $I1 = \{(-0.28, 0.96, 0.7)\}$
 - $I2 = \{0, 0.43, 0.7\}$
 - $I3 = \{1.31, 1.31, 0.7\}$
- Número
 - $m = 14$
 - $n = 22$
 - $r = -5$
- Punto
 - $A = (0, 0)$
 - $B = (0, 1)$

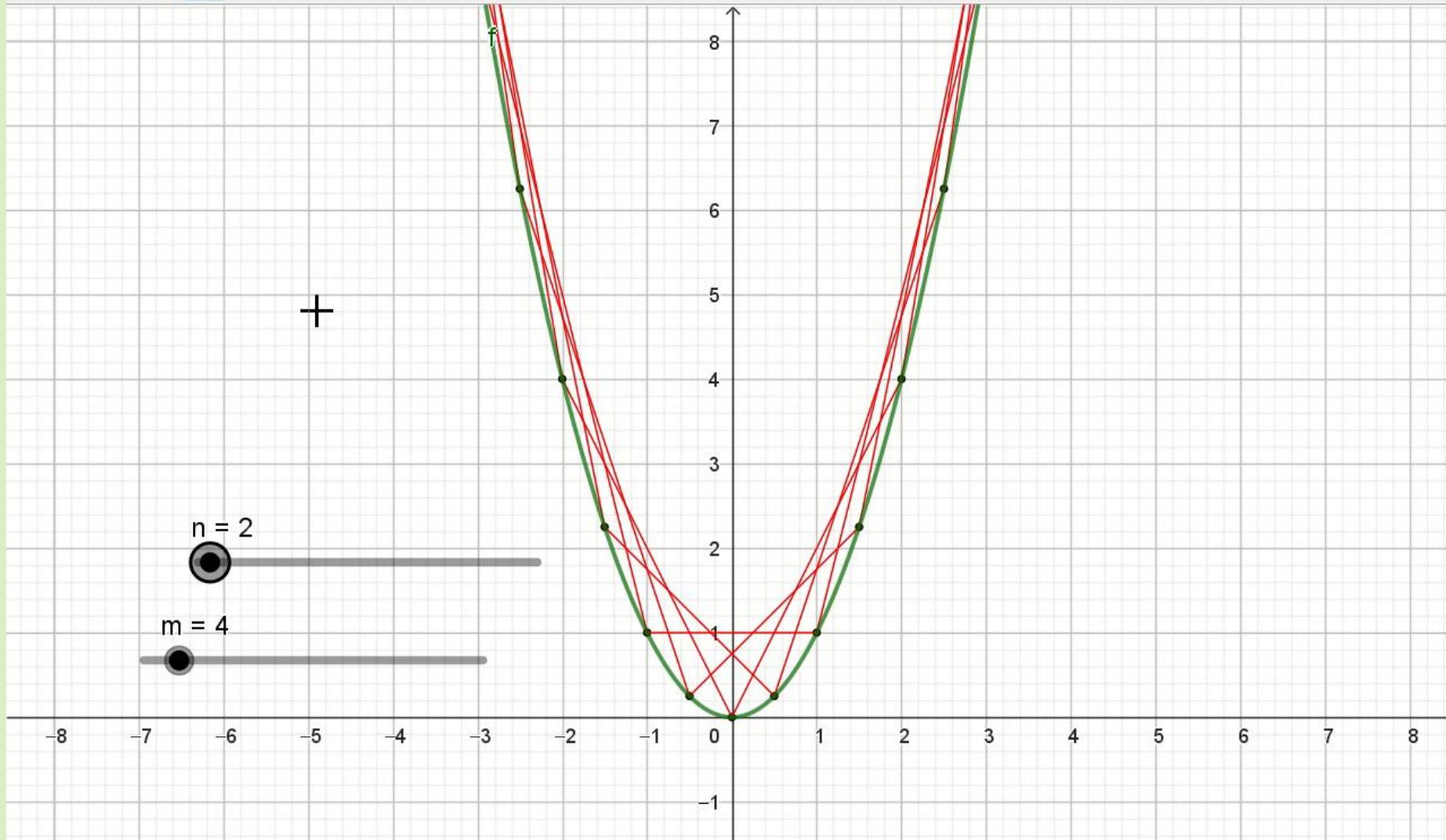
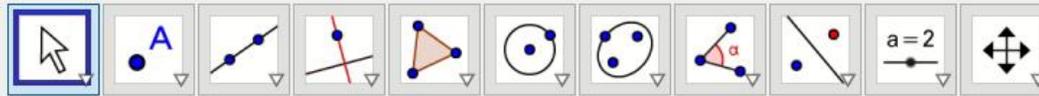
The 'Vista Gráfica' shows a circle with center A at (0,0) and point B at (0,1). A sequence of green segments connects points on the circle, starting from B and moving clockwise. A 'Redefine' dialog box is open over the diagram, with the following text:

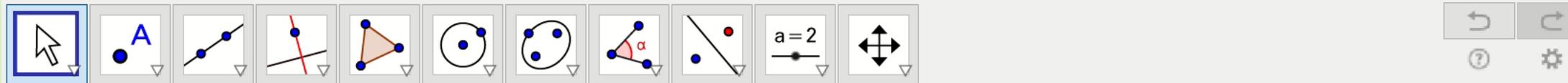
Redefine
Lista I2
Secuencia(Segmento(Rota(B, $k * 2\pi / n$), Rota(B, $1 / k * 2\pi / n$)), k, 1, n)

The dialog box has buttons for 'Propiedades...', 'OK', 'Cancela', and 'Aplicar'.

¿Y si elevamos al cuadrado?

The image shows a screenshot of the GeoGebra software interface. The window title is "circular3.ggb". The menu bar includes "Archivo", "Edita", "Vista", "Opciones", "Herramientas", "Ventana", and "Ayuda". The toolbar contains various geometric tools, including a selection tool, a point tool, a line tool, a perpendicular line tool, a triangle tool, a circle tool, a circle with center tool, an angle tool, a line with slope tool, a line with distance tool, and a move tool. The "Entrada:" field is empty. The left sidebar shows the "Vista Algebraica" and "Vista Gráfica" tabs. Under "Vista Algebraica", the "Cónica" section shows a circle $c: x^2 + y^2 = 1$. The "Lista" section shows three lists: $I1 = \{(-0.13, 0.99)$, $I2 = \{0, 0.27, 0.7$, and $I3 = \{0.66, 0.66,$. The "Número" section shows $m = 1$, $n = 47$, and $r = -5$. The "Punto" section shows $A = (0, 0)$ and $B = (0, 1)$. The main workspace shows a circle with a complex network of green lines connecting points on the circle. A "Redefine" dialog box is open, showing the formula: $\text{Secuencia}(\text{Segmento}(\text{Rota}(B, k * 2\pi / n), \text{Rota}(B, k^2 * 2\pi / n)), k, 1, n)$. The dialog box has buttons for "Propiedades...", "OK", "Cancela", and "Aplicar".





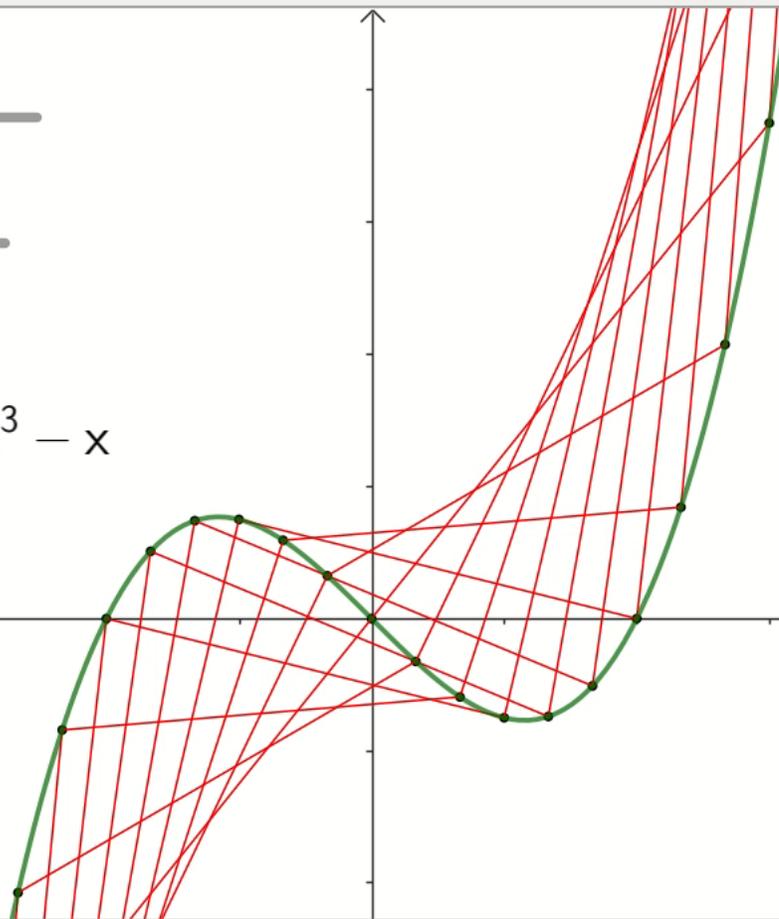
n = 6

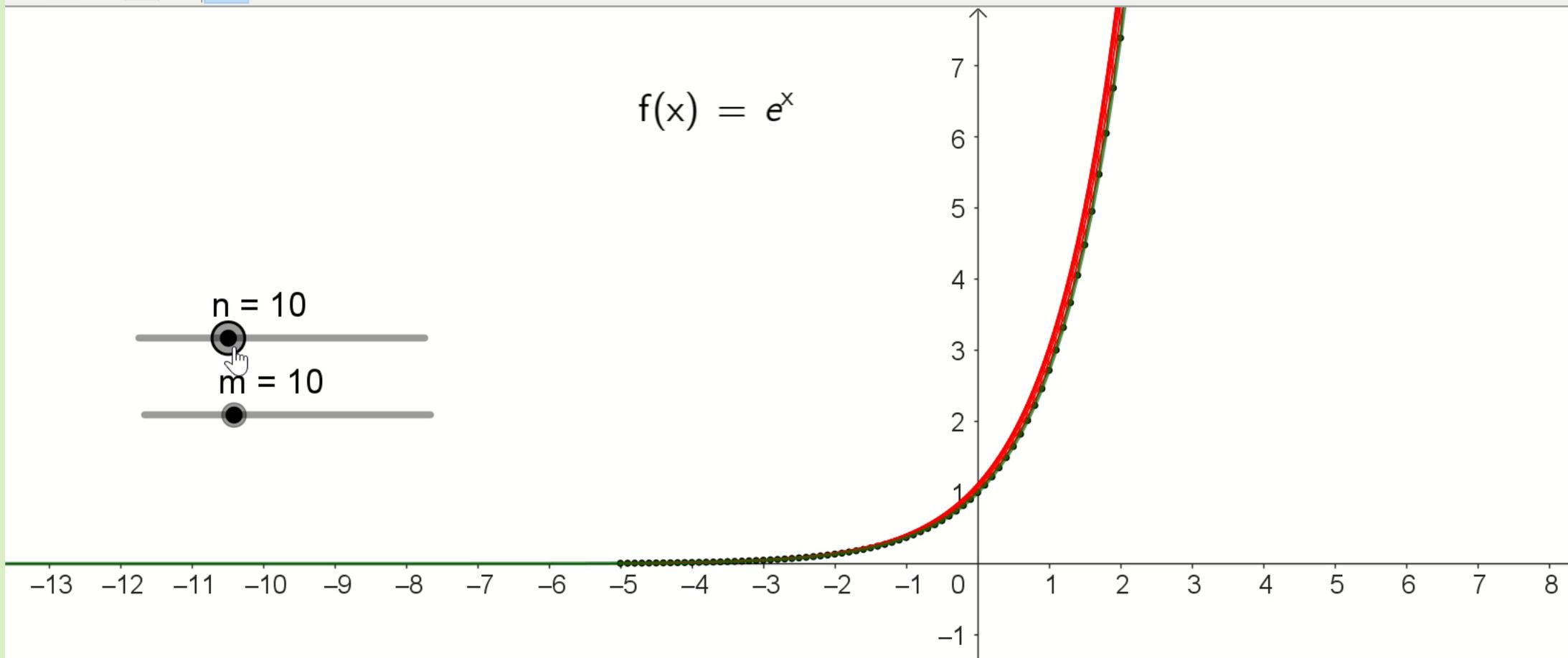
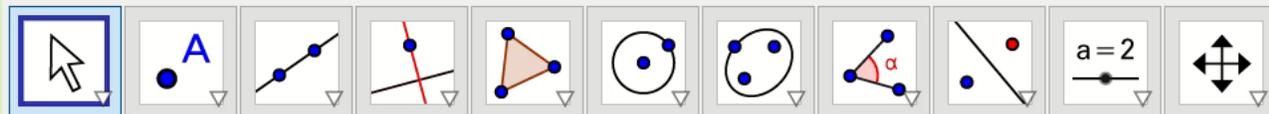


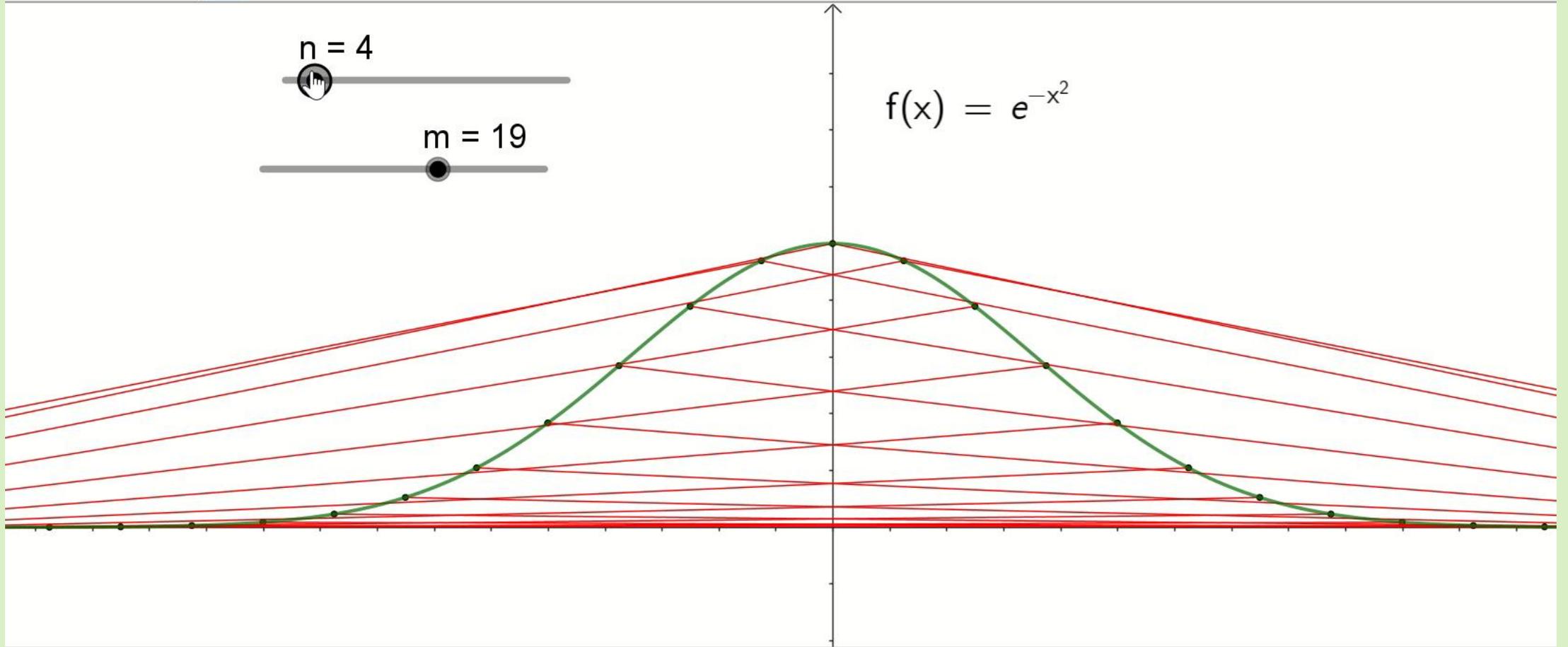
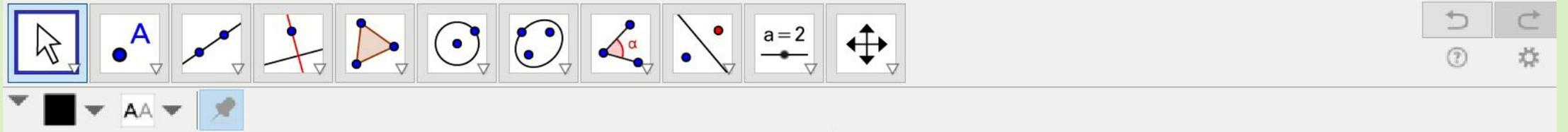
m = 9

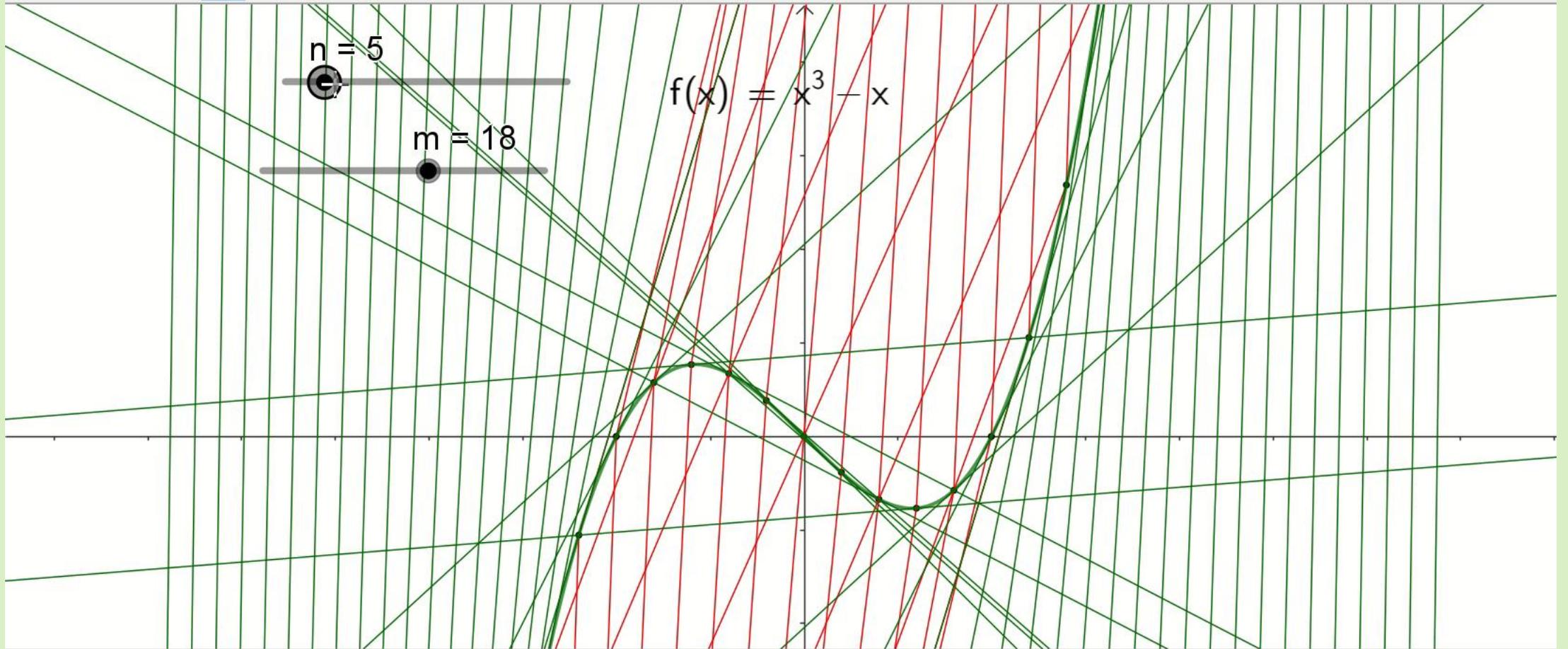
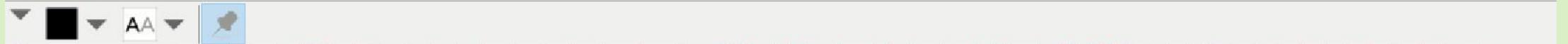
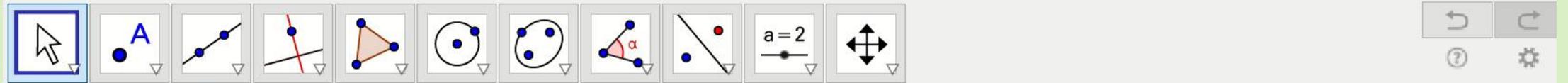


$$f(x) = x^3 - x$$

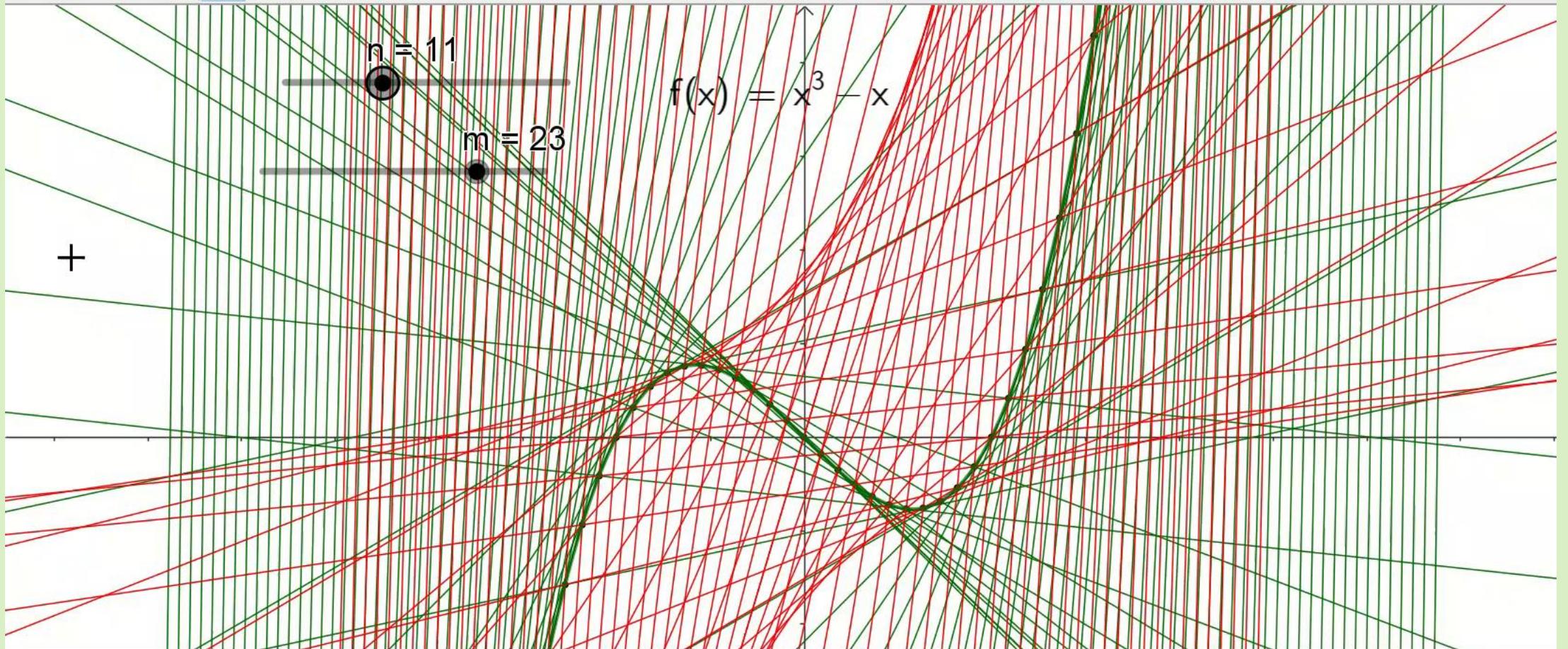
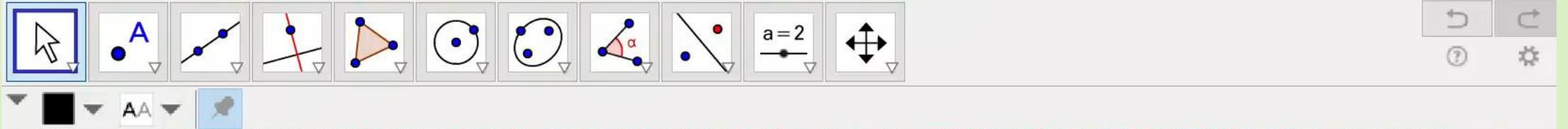








Archivo Edita Vista Opciones Herramientas Ventana Ayuda





¡Muchas gracias!

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